

Advanced Placement Calculus AB Summer Practice

Mr. Hulbert

The AP Calculus Course is a challenging, fast paced effort to prepare students for the AP Exam in May and also to provide a strong experience in preparation for any college calculus courses that may be taken after graduation. As such, the foundation upon which this course stands must be firmly in place. This practice is intended to assure that the mathematics covered in the past four years has been retained at a sufficient level so as to not hinder the development of new concepts with the pondering over fundamental computations.

This practice, with the exception of Section XI, is intended for the student to be able to complete independently, without the need to look up anything in either a text book, or on the internet. It should be noted, that anything the student does need to look up would indicate a need for study in that topic area. All of the material in this practice shall be needed at quick recall at some point during the calculus class. A need to look things up will hinder the student's process of understanding the concepts that will be addressed in this course. Therefore, one of the intentions of this practice is to reveal to the student topic areas that may require some independent remediation, hopefully before starting calculus. Section XI is, however, new material. This will be discussed in the first days of school, and students would benefit from preparation.

This practice is divided into two parts. The first is only informational; each section addresses some fundamental ideas and skills required for that topic. The second section consists of the problems that are to be solved. A reasonable amount of work should be shown for each question. With rare exceptions, an answer standing alone would not be accepted on a graded assesment. When in doubt, show some sort of work or description of your thought process. Answers are provided at the end of the packet, for self-assessment.

The last page is the course information sheet. Please have this signed and ready to hand in on the first day of school.

If you lose this practice, it will be posted on the Columbia High School Math Department website. Upon the opening of school in September, questions may be asked over the course of the first few class days, during the homework review portion of class. In addition, after school help will be available upon request during that time. **On Tuesday, September 12, 2023, there will be a quiz in class based on this material.**

Section A – Information

I. Polynomials & Equations

IDEAS

- | | | |
|---|---|---|
| <input checked="" type="checkbox"/> multiplying polynomials | <input checked="" type="checkbox"/> factoring trinomials | <input checked="" type="checkbox"/> factoring difference of 2 squares |
| <input checked="" type="checkbox"/> greatest common factors | <input checked="" type="checkbox"/> factoring sum/difference of cubes | <input checked="" type="checkbox"/> factor by grouping |
| <input checked="" type="checkbox"/> solving quadratics by factoring | <input checked="" type="checkbox"/> solving quadratics by quadratic formula | <input checked="" type="checkbox"/> solving quadratics by completing the square |
| <input checked="" type="checkbox"/> solve higher order polynomial equations algebraically | <input checked="" type="checkbox"/> solve radical equation | <input checked="" type="checkbox"/> use fractional exponents |
| <input checked="" type="checkbox"/> simplify complex fractions | <input checked="" type="checkbox"/> solve equations containing rational expressions | <input checked="" type="checkbox"/> calculate rational exponents without a calculator |

CALCULATOR SKILLS

- | | | |
|--|--|--|
| <input checked="" type="checkbox"/> graph polynomial equations | <input checked="" type="checkbox"/> solve equations graphically | <input checked="" type="checkbox"/> find roots |
| <input checked="" type="checkbox"/> find extrema | <input checked="" type="checkbox"/> use appropriate zoom and adjust window | <input checked="" type="checkbox"/> format and use the table |
| <input checked="" type="checkbox"/> find any y -value given an x -value, not using the table | | |

II. Exponentials & Logarithms

IDEAS

- | | | |
|--|--|--|
| <input checked="" type="checkbox"/> sketch graph of exponential function | <input checked="" type="checkbox"/> determine domain and range of exponential function | <input checked="" type="checkbox"/> sketch graph of logarithmic function |
| <input checked="" type="checkbox"/> determine domain and range of logarithmic function | <input checked="" type="checkbox"/> use properties of logarithms to manipulate log expressions | <input checked="" type="checkbox"/> change the base of a logarithmic expression |
| <input checked="" type="checkbox"/> solve exponential equations | <input checked="" type="checkbox"/> solving logarithmic equations | <input checked="" type="checkbox"/> switch between exponential form and logarithmic form |

CALCULATOR SKILLS

- | | | |
|---|--|---|
| <input checked="" type="checkbox"/> graph exponential equations | <input checked="" type="checkbox"/> graph logarithmic equations | <input checked="" type="checkbox"/> solve equations graphically |
| <input checked="" type="checkbox"/> use base e to calculate | <input checked="" type="checkbox"/> calculate logarithmic values | |

III. Graphing Polynomials and Rational Functions

i IDEAS

- | | | |
|--|--|---|
| <input checked="" type="checkbox"/> find slope of line given two points | <input checked="" type="checkbox"/> write equation of line in slope intercept form | <input checked="" type="checkbox"/> write equation of line in point intercept form |
| <input checked="" type="checkbox"/> recognize intervals of increase or decrease on a graph | <input checked="" type="checkbox"/> determine turning points or vertices of graphs | <input checked="" type="checkbox"/> determine end behavior of a graph from the equation |
| <input checked="" type="checkbox"/> find vertical and horizontal asymptotes algebraically | <input checked="" type="checkbox"/> find x and y intercepts algebraically | <input checked="" type="checkbox"/> use long division to find factors and roots |
| <input checked="" type="checkbox"/> determine algebraically and graphically if a function is even or odd | | <input checked="" type="checkbox"/> use synthetic division to find factors and roots |

📊 CALCULATOR SKILLS

- | | | |
|--|--|--|
| <input checked="" type="checkbox"/> graph polynomial equations | <input checked="" type="checkbox"/> graph polynomial equations | <input checked="" type="checkbox"/> find roots |
| <input checked="" type="checkbox"/> find extrema | | <input checked="" type="checkbox"/> format and use the table |
| <input checked="" type="checkbox"/> find any y -value given an x -value, not using the table | <input checked="" type="checkbox"/> use appropriate zoom and adjust window | |

IV. Transformations of Graphs

i IDEAS

- | | |
|---|---|
| <input checked="" type="checkbox"/> transformation rules for translation, reflection, scaling | <input checked="" type="checkbox"/> parent graphs of the following: |
| <input checked="" type="checkbox"/> complete the square on quadratic to find vertex | |

$$f(x) = x^2$$

$$f(x) = \log x$$

$$f(x) = \sqrt{x}$$

$$f(x) = \ln x$$

$$f(x) = b^x$$

$$f(x) = |x|$$

$$f(x) = \frac{1}{x}$$

📊 CALCULATOR SKILLS

- | | |
|---|--|
| <input checked="" type="checkbox"/> graph equations | <input checked="" type="checkbox"/> use multiple graphs simultaneously |
|---|--|

V. Conic Sections

i IDEAS

- | | |
|--|--|
| <input checked="" type="checkbox"/> recognize standard form of equation for parabola, circle, ellipse, hyperbola | |
| <input checked="" type="checkbox"/> sketch a graph using information from equation to determine features such as vertices, foci, axis of symmetry, major axis, minor axis, transverse axis, conjugate axis | |
| <input checked="" type="checkbox"/> complete the square to put equation in standard form | <input checked="" type="checkbox"/> write an equation based on the graph |

📊 CALCULATOR SKILLS

- ☒ graph using multiple equations ☒ solve equations graphically

VI. Trigonometry Basics

i IDEAS

- ☒ convert degrees \leftrightarrow radians ☒ know common angles in radians ☒ signs of trig functions in the different quadrants (ASTC)
- $\left(90^\circ = \frac{\pi}{2}, 45^\circ = \frac{\pi}{4}, 30^\circ = \frac{\pi}{6}, \text{etc.} \right)$
- ☒ 21 club values \rightarrow **NO CALC!!!** ☒ reference angles (degrees/radians) ☒ reciprocal functions

	$0^\circ (0)$	$30^\circ (\pi/6)$	$45^\circ (\pi/4)$	$60^\circ (\pi/3)$	$90^\circ (\pi/2)$	$180^\circ (\pi)$	$270^\circ (3\pi/2)$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}$	1	0	-1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1	0
$\tan\theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	undef	0	undef

📊 CALCULATOR SKILLS

- ☒ use appropriate mode ☒ convert degrees \leftrightarrow radians in a rational result (with π)
- ☒ calculate trig values ☒ appropriate use of inverse functions

VII. Trigonometry Graphs

i IDEAS

- ☒ amplitude ☒ frequency (including π) ☒ period (including rational)
- ☒ transformations of basic graphs ☒ work in radians ☒ write equations based on graphs
- ☒ graphs of all 6 trig functions

📊 CALCULATOR SKILLS

- ☒ graph trig functions ☒ work in radians ☒ format and use the table
- ☒ work with multiple equations simultaneously ☒ use appropriate zoom and adjust window

VII. Trigonometric Identities

i IDEAS

- ☒ Pythagorean identities ☒ double angle identities ☒ reciprocal identities
- ☒ quotient identities ☒ be able to substitute and manipulate trig identities to simplify expressions

IX. Trigonometric Equations

IDEAS

- ☒ solve algebraically using identities
- ☒ work with frequencies other than 1
- ☒ work in various intervals

CALCULATOR SKILLS

- ☒ solve graphically
- ☒ work in radians
- ☒ format and use the table
- ☒ use appropriate zoom and adjust window

X. Geometry

IDEAS

- ☒ special right triangles → 30–60–90 and 45–45–90
- ☒ area formulae for: triangle, rectangle, square, parallelogram, trapezoid, circle
- ☒ volume formulae for: rectangular prism, pyramid, cylinder, cone, sphere

XI. Counting Principles and Binomial Expansion

This is a new topic. Do your best to understand it. Khan Academy has some videos that address it. We will discuss this when you return to school, under the assumption that you have attempted to understand.

IDEAS

- ☒ counting principal → how many ways to do something.

Multiply the number of options for each choice:

A woman has 8 blouses, 5 skirts and 6 scarves. How many outfits can be assembled from these?

$$\frac{8}{\text{blouses}} \cdot \frac{5}{\text{skirts}} \cdot \frac{6}{\text{scarves}} = 240 \text{ outfits}$$

A license plate consists of consists of 3 letter and 3 numbers. How many different plates possible?

$$\frac{26}{\text{letter}} \cdot \frac{26}{\text{letter}} \cdot \frac{26}{\text{letter}} \cdot \frac{10}{\text{number}} \cdot \frac{10}{\text{number}} \cdot \frac{10}{\text{number}} = 26^3 \cdot 10^3 = 17,576,000$$

- ☒ ordered arrangements → putting a whole group in order

Putting 3 people in order in a line: ABC, ACB, BAC, BCA, CAB, CBA

Counting principle: $\frac{3}{\text{# available for 1st spot}} \cdot \frac{2}{\text{# available for 2nd spot}} \cdot \frac{1}{\text{# available for 3rd spot}} = 6$

Uses a function called *factorial*, whose symbol is after the # → this is 3!

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 \quad 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \quad n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$$

***Fact:** $0! = 1$...weird because of definitions

☑ ordered arrangements → putting part of a group in order

Taking 5 people, selecting 3 to stand in order in a line

People ABCDE → in line ABC, BCD, CDE, AEB, DCA, etc.

Counting principle: $\frac{5}{\text{\# available for 1st spot}} \cdot \frac{4}{\text{\# available for 2nd spot}} \cdot \frac{3}{\text{\# available for 3rd spot}} = 60$

This is a function called a *permutation*, whose symbol is ${}_nP_r$, where n = total number of objects to choose from, and r = number of objects chosen. This one is ${}_5P_3$.

***Fact:** choosing all the items is the same as the factorial → ${}_5P_5 = 5!$

☑ arrangements where order doesn't matter → selecting committees or teams

Taking 5 people, selecting 3 to represent the group (doesn't matter who was chosen 1st)

People ABCDE → ABC is same as BAC, CBA, BCA, etc.

Counting principle hard to apply here

This is a function called a *combination*, whose symbol is ${}_nC_r$, where n = total number of objects to choose from, and r = number of objects chosen. This one is ${}_5C_3$.

Combinations can be calculated using permutations and factorials: ${}_nC_r = \frac{{}_nP_r}{(n-r)!}$

$${}_5C_3 = \frac{{}_5P_3}{(5-3)!} = \frac{5 \cdot 4 \cdot 3}{2!} = \frac{5 \cdot 4 \cdot 3}{2 \cdot 1} = 30$$

***Facts:** There is only one way of choosing none of the items → ${}_nC_0 = 1$

There is only one way of choosing all of the items → ${}_nC_n = 1$

There are n ways to choose one object → ${}_nC_1 = n$

There are n ways to choose all but one object → ${}_nC_{n-1} = n$

There are pairs of choosing items that are the same → ${}_nC_r = {}_nC_{n-r}$

↙ ↘
Add up to n

Examples: ${}_5C_3 = {}_5C_2 = 30$; ${}_8C_3 = {}_8C_5 = 56$; ${}_{100}C_1 = {}_{100}C_{99} = 100$

☑ expanding a binomial raised to a power

$$(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$$

$$(x+y)^3 = (x+y)(x+y)(x+y) = x^3 + 3x^2y + 3xy^2 + y^3 \rightarrow \text{any power higher than 3 is difficult and tedious}$$

Binomial Expansion → uses known patterns and combinations as a shortcut

$$(x+y)^n = {}_nC_0(x)^n(y)^0 + {}_nC_1(x)^{n-1}(y)^1 + {}_nC_2(x)^{n-2}(y)^2 + {}_nC_3(x)^{n-3}(y)^3 + \dots + {}_nC_{n-1}(x)^1(y)^{n-1} + {}_nC_n(x)^0(y)^n$$

$$\begin{aligned}(x+y)^5 &= {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^4(y)^1 + {}_5C_2(x)^3(y)^2 + {}_5C_3(x)^2(y)^3 + {}_5C_4(x)^1(y)^4 + {}_5C_5(x)^0(y)^5 \\ &= 1x^5 + 5x^4y + 30x^3y^2 + 30x^2y^3 + 5xy^4 + 1y^5\end{aligned}$$

$$\begin{aligned}(x+2)^4 &= {}_4C_0(x)^4(2)^0 + {}_4C_1(x)^3(2)^1 + {}_4C_2(x)^2(2)^2 + {}_4C_3(x)^1(2)^3 + {}_4C_4(x)^0(2)^4 \\ &= 1x^4 + 4x^3(2) + 6x^2(2)^2 + 4x(2)^3 + 1(2)^4 \\ &= x^4 + 8x^3 + 24x^2 + 32x + 16\end{aligned}$$

$$\begin{aligned}(x-3)^3 &= {}_3C_0(x)^3(-3)^0 + {}_3C_1(x)^2(-3)^1 + {}_3C_2(x)^1(-3)^2 + {}_3C_3(x)^0(-3)^3 \\ &= 1x^3 + 3x^2(-3) + 3x^1(-3)^2 + 1(-3)^3 \\ &= x^3 - 9x^2 + 27x - 27\end{aligned}$$

📊 CALCULATOR SKILLS

☑ all 3 functions (factorials, permutations, combinations) can be calculated on the graphing calculator

Older operating systems:

MATH → → → PROB

Choices 2, 3, and 4

Have to enter numbers in proper order

For each individual function

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

4!	24
5 nPr 2	
5 nCr 3	10

Newer operating systems:

ALPHA → WINDOW

Choices 7, 8, and 9

Enter factorial number before

For Permutation and combinations

fill numbers into blanks

```
NORMAL FLOAT AUTO REAL RADIAN MP
1:abs(
2:summation Σ(
3:nDeriv(
4:fnInt(
5:logBASE(
6:√
7:nPr
8:nCr
9:!
FRAC FUNC MTRX YVAR
```

4!	24
${}_5P_2$	20
${}_5C_3$	10

Section B – Problems

I. Polynomials & Equations

Factor the following polynomials completely:

1. $6x^2 - 13x + 2$

2. $9x^2 - 12x + 4$

3. $5x^3 - x^2 - 5x + 1$

4. $x^3 + 3x^2 - x - 3$

5. $2x^3 + 13x^2 + 15x$

6. $4x^3 - 10x^2 + 6x$

7. $x^4 - 12x^2 + 27$

8. $9 - 9(x + 2)^2$

9. $3x^3 - 12x^2 + 6x - 24$

10. $5x^4 - 5y^4$

11. $5x^2 + 22x - 15$

Calculate without a calculator

12. $32^{\frac{3}{5}}$

13. $64^{\frac{2}{3}}$

14. $(-8)^{\frac{5}{3}}$

15. $\left(\frac{9}{4}\right)^{-\frac{5}{2}}$

Solve the following equations algebraically for x :

16. $6x^2 + x - 12 = 0$

17. $x^3 = x$

18. $x^4 - 3x^2 + 2 = 0$

19. $2x^3 - 6x^2 - 6x + 18 = 0$

20. $\sqrt{2x+7} - x = 2$

21. $\sqrt{2x+6} - \sqrt{x+4} = 1$

$$22. \quad (x+1)^{\frac{2}{3}} = 4$$

$$23. \quad (3-x)^{\frac{4}{3}} + 2 = 18$$

$$24. \quad 8(2x+1)^{\frac{3}{2}} = 27$$

$$25. \quad \frac{x}{3} + \frac{3x}{4} = 2$$

$$26. \quad \frac{2}{x} = \frac{3}{x-2} - 1$$

$$27. \quad \frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$$

Solve by completing the square:

$$28. \quad x^2 + 6x - 5 = 0$$

Solve by the quadratic formula:

$$29. \quad 2x^2 + 3x - 1 = 0$$

II. Exponentials & Logarithms

1. Write the expression $\log_2 \sqrt{\frac{x^2}{y}}$ in terms of $\log_2 x$ and $\log_2 y$.
2. Write as the logarithm of a single quantity: $\frac{1}{4} \log_b 16 - 2 \log_b 5 + \log_b 7$
3. Find the domain of $f(x) = \log(x - 2)$.
4. If $A = A_0 e^{rt}$, find $\ln A$ in simplest form.
5. Solve for x , rounding to the nearest hundredth:
a) $4 + e^x = 6.72$ b) $y = \log_3 7$ c) $\log_5(x + 3) - \log_5 x = \log_5 4$

d) $\frac{1}{2} \ln x = 4 \ln 2 - \ln 4$ e) $2^{3x} = 7$ f) $e^x = 7$

III. Graphing Polynomials and Rational Functions

1. Determine the left-hand and right-hand behavior of the graphs of the following functions:

a) $f(x) = -x^5 + 2x^2 - 1$ b) $f(x) = 2x^3 + 5x^2 - 8$ c) $f(x) = -7x^4 + \frac{1}{2}x^3 - 3$ d) $f(x) = 4x^6 + 8x^5 - 2x^2 + 4$

2. Determine all zeros, asymptotes, and intercepts of the graph of

a) $f(x) = \frac{6x+3}{x^2-1}$ b) $f(x) = \frac{x^2-4}{x^2-6x+5}$

3. Divide using long division $(2x^5 - 3x^3 + 5x^2 - 1) \div (2x + 3)$

4. Divide using synthetic division $\frac{2x^4 - 5x^3 + 7x^2 - 3x + 1}{x - 3}$

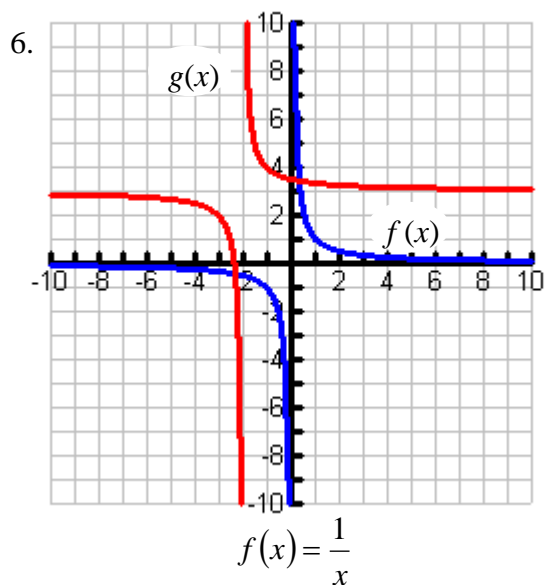
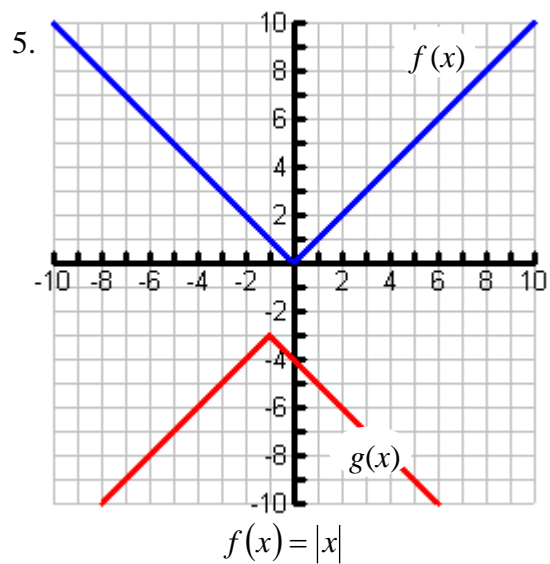
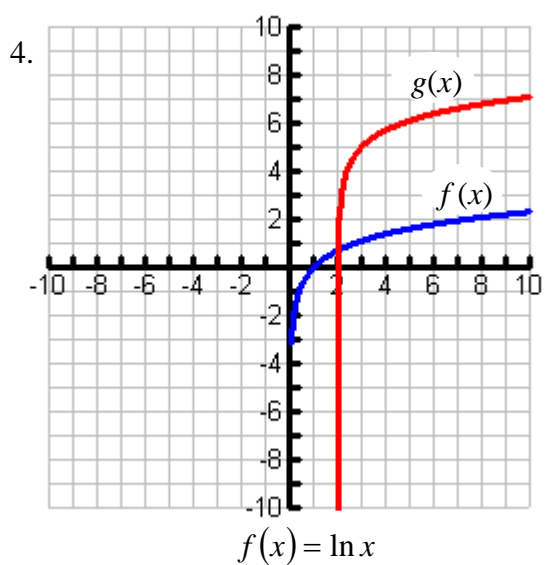
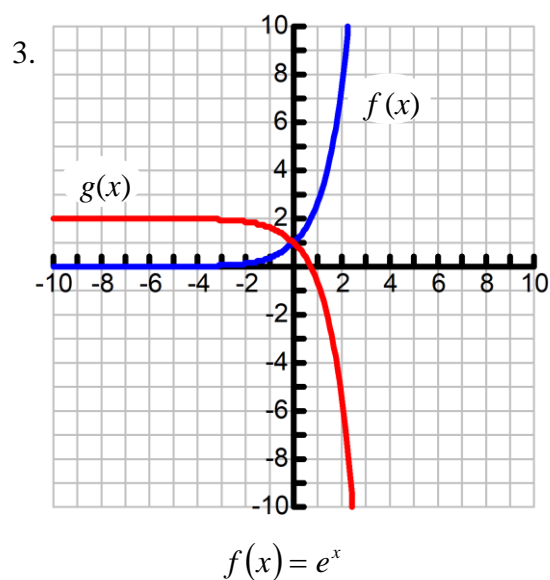
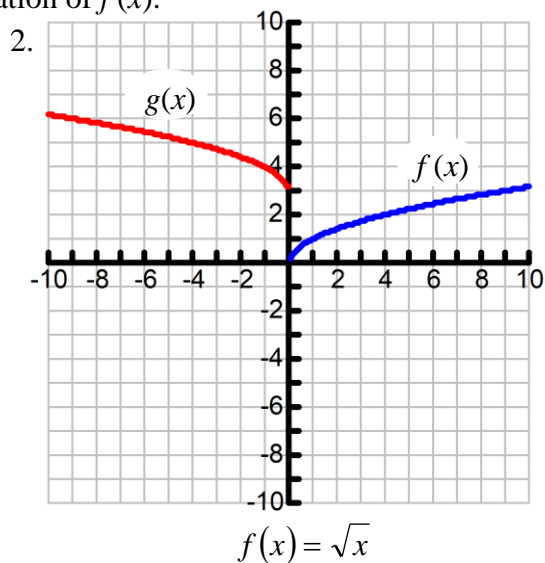
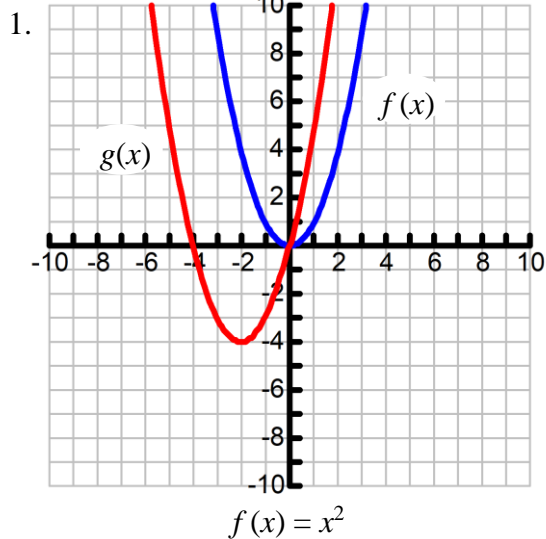
5. a) Determine if $x - 3$ a factor of $x^3 - x^2 - 14x + 1$. b) Determine if $x - 1$ a factor of $2x^3 + x^2 - 5x + 2$

6. Determine if the following functions are even, odd, or neither...justify your answer.

a) $f(x) = 3x^4 - 6x^2$ b) $f(x) = -x^6 - 2x^4$ c) $f(x) = x^{\frac{2}{3}}$ d) $f(x) = x\sqrt{x-3}$

IV. Transformations of Graphs

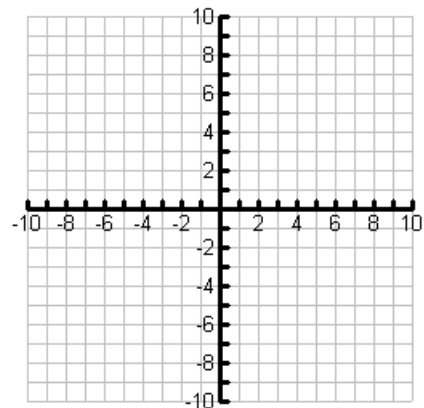
Write the equation of $g(x)$, given the graph and equation of $f(x)$.



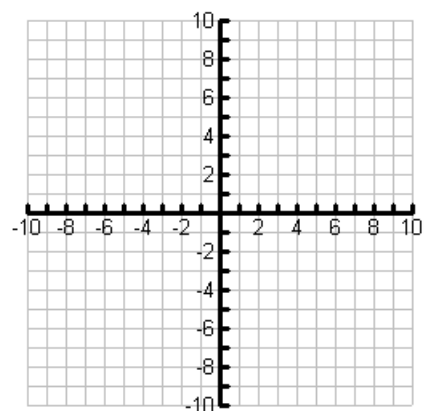
V. Conic Sections

1. Write the equation in the standard form for its type and sketch a graph:

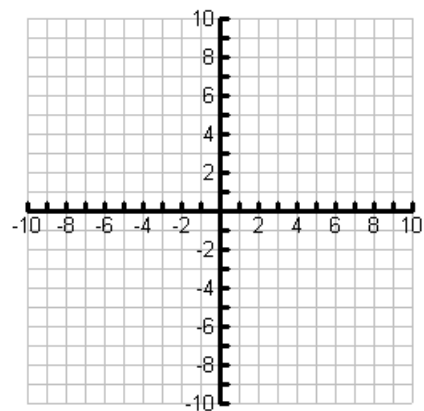
a) $y^2 - 10y + 12x + 37 = 0$



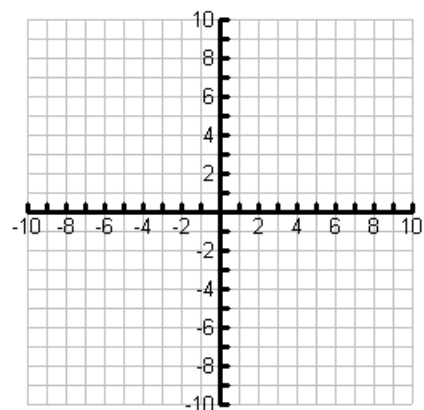
b) $9x^2 - 36x + y^2 + 14y = -76$



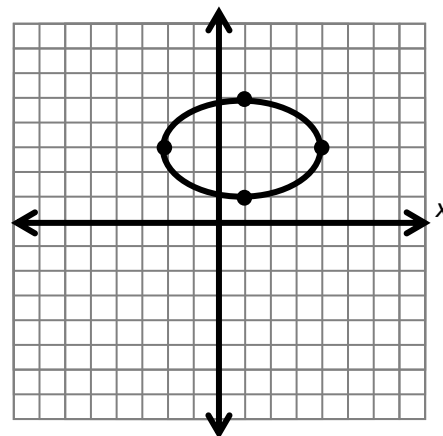
c) $3x^2 + 6x - 2y^2 - 12y = 21$



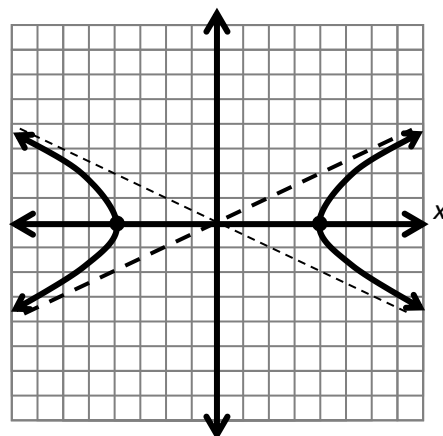
d) $x^2 + y^2 - 4x + 8y + 14 = 0$



2. Write the standard form of the equation for the ellipse shown here:



3. Write the standard form of the equation for the hyperbola shown here:



VI. Trigonometry Basics

1. Convert the degree measure to radian measure, in terms of π :

a) 45° b) 90° c) 30° d) 60° e) 270° f) 50° g) 24°

2. Convert the radian measure to degree measure:

a) $\frac{2\pi}{3}$ b) $\frac{5\pi}{4}$ c) $\frac{11\pi}{6}$ d) $\frac{2\pi}{5}$

3. Determine the quadrant in which the terminal side of θ lies:

a) If $\cos \theta > 0$ and $\sin \theta < 0$ b) if $\sec \theta < 0$ and $\cot \theta > 0$ c) if $\csc \theta > 0$ and $\tan \theta < 0$

4. Determine the EXACT function value, in simplest form, without a calculator:

a) $\cos \frac{\pi}{3}$ b) $\tan \frac{5\pi}{6}$ c) $\sin \frac{\pi}{4}$ d) $\cot \frac{2\pi}{3}$ e) $\csc \frac{11\pi}{6}$ f) $\sec \frac{7\pi}{4}$

VII. Trigonometric Graphs

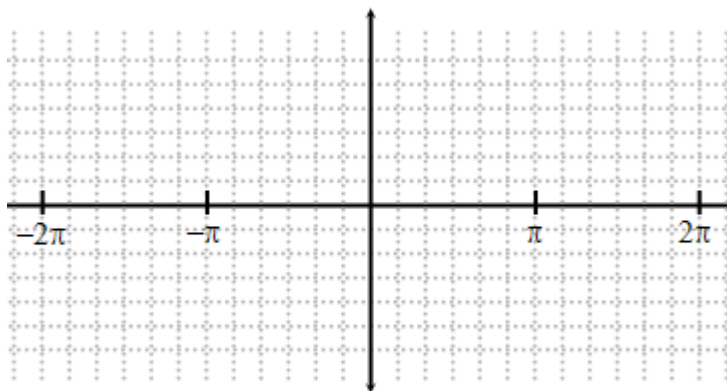
1. Find the amplitude, period and frequency for each equation:

a) $y = 3 \sin 2x$ b) $f(x) = -2 \cos \frac{1}{2}x$ c) $y = 9 \tan 3\theta$

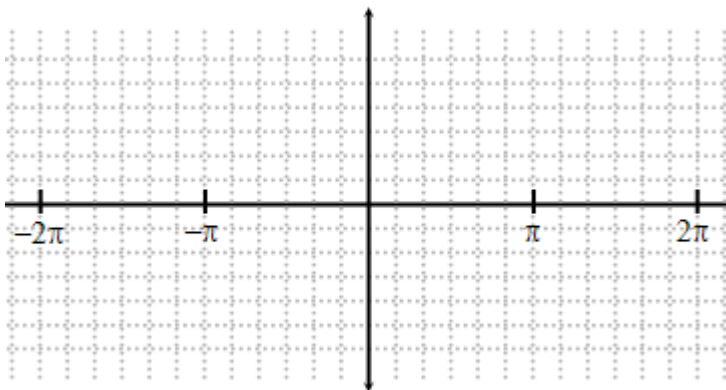
d) $s(t) = -\frac{3}{4} \sin \frac{\pi}{12}t$ e) $f(\theta) = \frac{\pi}{2} \cos 4\pi\theta$ f) $y = \frac{2}{3} \tan \frac{\pi x}{2}$

2. Sketch a graph by hand, then verify with a graphing calculator

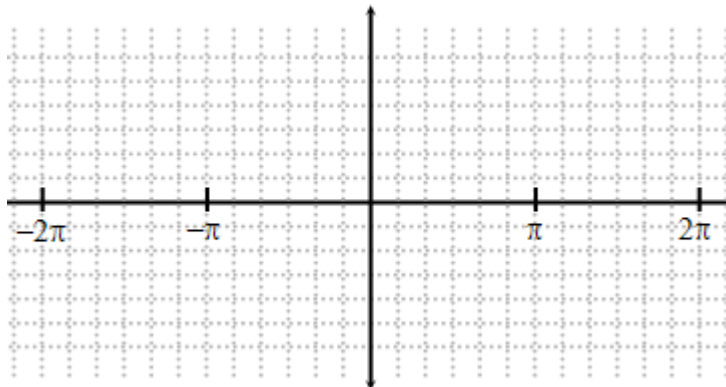
a) $y = 2 \sin 3\left(x + \frac{\pi}{3}\right) - 1$ on $(-2\pi, 2\pi)$



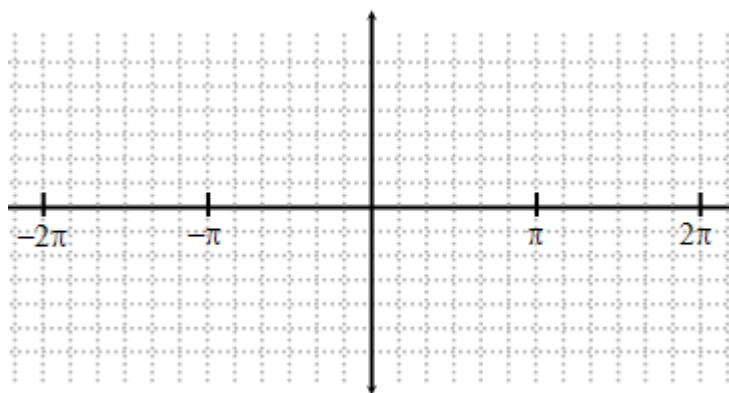
b) $y = -3 \cos \frac{1}{2}\left(x - \frac{\pi}{6}\right) + 1$ on $(-2\pi, 2\pi)$



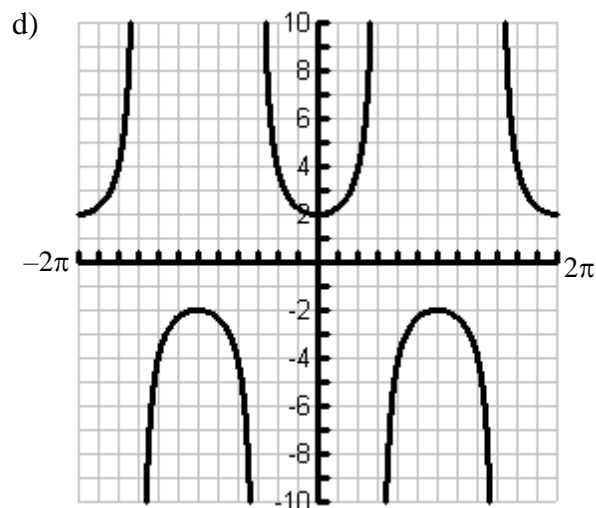
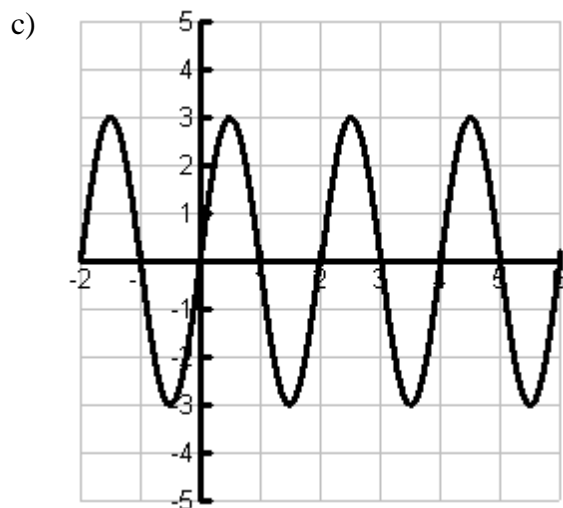
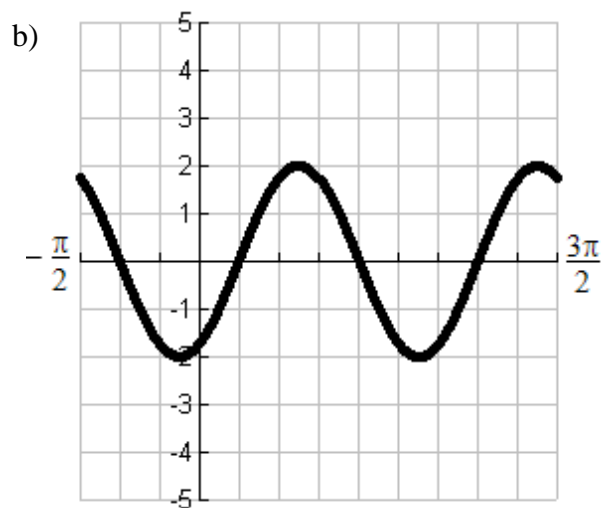
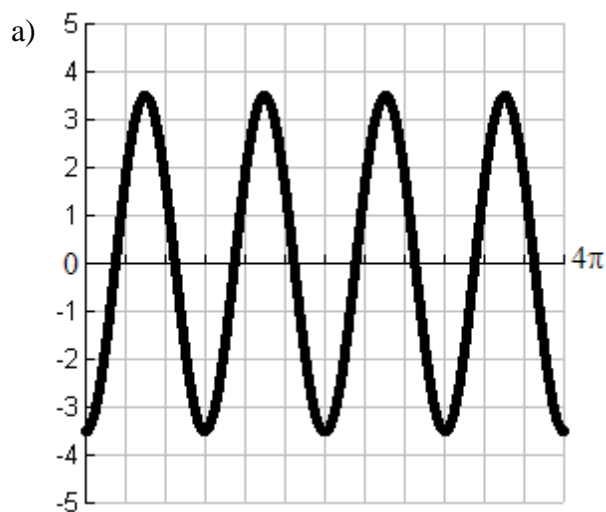
c) $y = \tan \frac{x}{2}$ on $(-2\pi, 2\pi)$



d) $y = 2\csc x$ on $(-2\pi, 2\pi)$



3. Write the equation for the given graph, using the smallest horizontal shift possible.



VIII. Trigonometric Identities

Verify the following identities:

$$1. \quad \frac{2 \cos x}{\sin 2x} = \csc x$$

$$2. \quad \frac{\cos^2 x}{1 + \sin x} = 1 - \sin x$$

$$3. \quad \sec^2 x (\cos 2x) = 2 - \sec^2 x$$

$$4. \quad \csc^2 x (\sec^2 x - 1) = \sec^2 x$$

$$5. \quad 1 - \cos 2x + 2 \cos^2 x = 2$$

IX. Trigonometric Equations

Find exact values of all solutions in the interval stated for each. Round to the nearest hundredth if exact value not possible.

1. $\sec^2 x - 2 \tan x = 4$ on $[0, 2\pi)$

2. $4 \sin x \cos x = \sqrt{2}$ on $[0, 2\pi)$

3. $6 \cos^2(2x) - 3 = 0$ on $[0, 2\pi)$

4. $\sin 3x = 0$ on $[-\pi, \pi]$

5. $4 \cos^2\left(\frac{\pi}{3}x\right) - 3 = 0$ on $[0, 3\pi]$

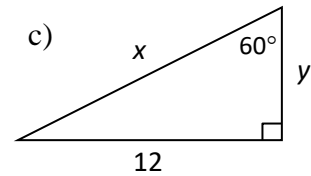
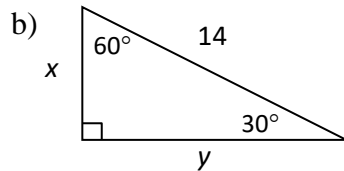
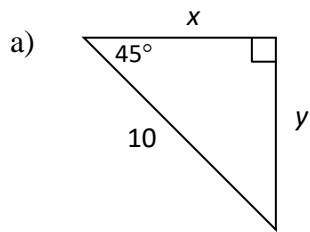
6. $(\sin 2x + \cos 2x)^2 = 1$ on $[0, 2\pi)$

7. $3 \tan(\pi x) - 3 = 0$ on $[0, 2\pi)$

8. $2 \csc^3\left(\frac{\pi}{4}x\right) - 10 = 6$ on $[0, 4\pi]$

X. Geometry

1. Find x and y in simplest radical form:

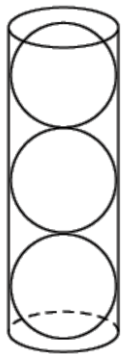


2. The circumference of a circle is 84π . Find its area.

3. Derive the formula for the area of an equilateral triangle whose side is s . Find the exact area of an equilateral triangle whose side is $12\sqrt{3}$ ft.

4. An isosceles trapezoid has legs which measure 10 inches, and bases that measure 5 and 17 inches, respectively. Find the area of this trapezoid.

5. A cone has a radius of 3 and a volume of 12π . A similar cone has a height of 8. Find the volume of the second cone, in terms of π .
6. A rectangular prism has a length, width, height ratio of $\frac{7}{2} : 4 : 5$. If it has a volume of 560 m^3 , find its surface area.
7. Three tennis balls are packed so that they touch each end and the sides of their can exactly. If the volume of the can is 708.6561235 cm^3 , find the volume of a single tennis ball, to the nearest tenth of a cm^3 .



XI. Counting Principles and Binomial Expansion

1. Megan decides to go out to eat. The menu at the restaurant has four appetizers, three soups, seven entrées and five desserts. If Megan decides to order an appetizer *or* a soup, one entrée, and one dessert, how many different choices can she make?
2. One state issues license plates with 3 letters and 4 numbers, allowing only numbers to repeat. Another state uses the same 3 letter, 4 number set up, but only allows repetition of the letters. Which state will have more available plates, and by how many?
3. 12 people run in a race. Find the following:
 - a. The number of ways they can finish the race.
 - b. The number of arrangements of a 1st, 2nd, and 3rd place finish.
 - c. The number of ways you can randomly select any 3 participants for a drug test.
4. Expand the following, using the binomial expansion:
 - a. $(x + 1)^3$
 - b. $(x - 2)^3$
 - c. $(3x - 2)^3$
 - d. $(x^2 - 4)^3$
 - e. $(x - 2)^4$
 - f. $(2x + 3)^4$
 - g. $(x + 2)^5$

Advanced Placement Calculus AB Summer Practice Answers

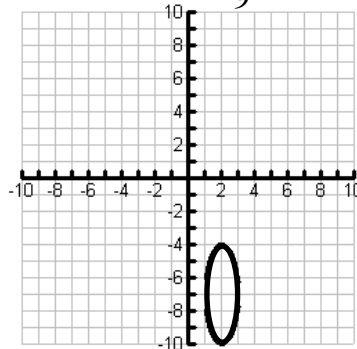
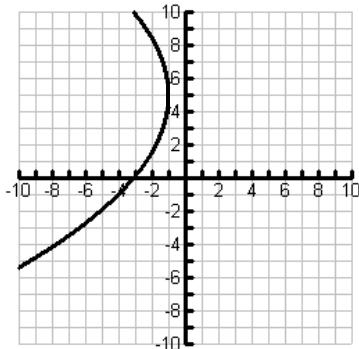
- I.** 1. $(6x-1)(x-2)$ 2. $(3x-2)(3x-2)$ 3. $(5x-1)(x+1)(x-1)$ 4. $(x+3)(x+1)(x-1)$
 5. $x(2x+3)(x+5)$ 6. $2x(2x-3)(x-1)$ 7. $(x+3)(x-3)(x+\sqrt{3})(x-\sqrt{3})$ 8. $-9(x+3)(x+1)$
 9. $3(x^2+2)(x-4)$ 10. $5(x^2+y^2)(x+y)(x-y)$ 11. $(5x-3)(x+5)$ 12. 8 13. $\frac{1}{16}$ 14. -32 15. $\frac{32}{243}$
 16. $\left\{-\frac{3}{2}, \frac{4}{3}\right\}$ 17. $\{-1, 0, 1\}$ 18. $\{-\sqrt{2}, -1, \sqrt{2}\}$ 19. $\{-\sqrt{3}, 3, \sqrt{3}\}$ 20. $\{1\}$ 21. $\{5\}$ 22. $\{-9, 7\}$
 23. $\{-5, 11\}$ 24. $\left\{-\frac{5}{18}\right\}$ 25. $\left\{\frac{24}{13}\right\}$ 26. $\{-1, 4\}$ 27. $\{ \}$ 28. $\{-3 \pm \sqrt{14}\}$ 29. $\left\{\frac{-3 \pm \sqrt{17}}{4}\right\}$

- II.** 1. $\log_2 x - \frac{1}{2} \log_2 y$ 2. $\log_b \frac{14}{25}$ 3. $\{x : x > 2\}$ or $(2, \infty)$ 4. $\ln A_0 + rt$
 5. a) 1.00 b) 1.77 c) 1.00 d) 16.00 e) 0.94 f) 1.95

- III.** 1. a) rises left, falls right b) falls left, rises right c) falls left, falls right d) rises left, rises right
 2. a) zeros $x = -\frac{1}{2}$; vertical asymptotes $x = \pm 1$; horizontal asymptote $y = 0$; y-int $y = -3$
 b) zeros $x = \pm 2$; vertical asymptotes $x = 1, 5$; horizontal asymptote $y = 1$; y-int $y = -\frac{4}{5}$
 3. $x^4 - \frac{3}{2}x^3 + \frac{3}{4}x^2 + \frac{11}{8}x - \frac{33}{16} + \frac{83}{16(2x+3)}$ 4. $2x^3 + x^2 + 10x + 27 + \frac{82}{x-3}$ 5. a) no b) yes
 6. a) even b) even c) even d) neither

- IV.** 1. $g(x) = (x+2)^2 - 4$ 2. $g(x) = \sqrt{-x} + 3$ 3. $g(x) = -e^x + 2$ 4. $g(x) = \ln(x-2) + 5$
 5. $g(x) = -|x+1| - 3$ 6. $g(x) = \frac{1}{x+2} + 3$

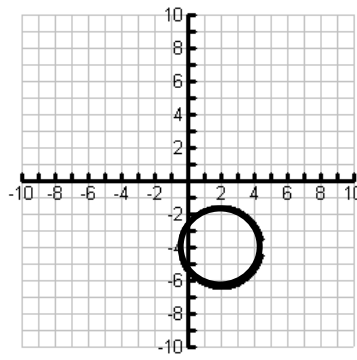
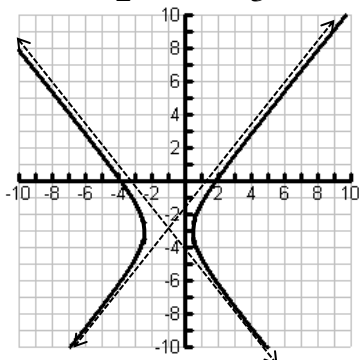
- V.** 1. a) $(y-5)^2 = -12(x+1)$ b) $(x-2)^2 + \frac{(y+7)^2}{9} = 1$ 2. $\frac{(x-1)^2}{9} + \frac{(y-3)^2}{4} = 1$



$$3. \frac{x^2}{16} - \frac{y^2}{4} = 1$$

$$c) \frac{(x+1)^2}{2} - \frac{(y+3)^2}{3} = 1$$

$$d) (x-2)^2 + (y+4)^2 = 6$$

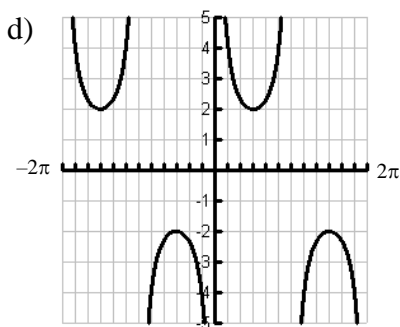
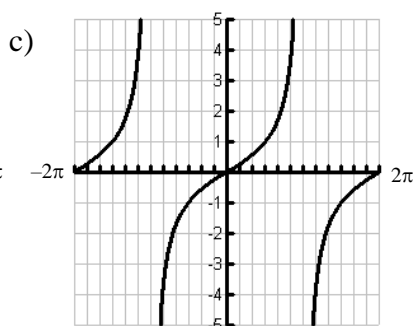
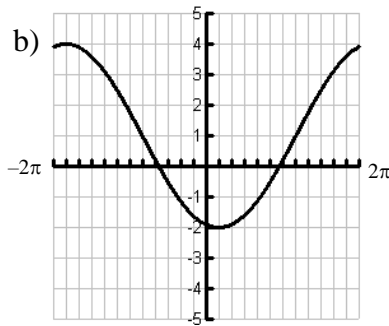
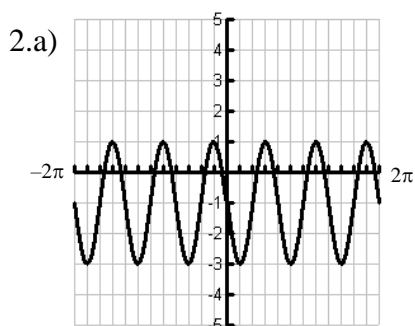


VI. 1.a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{3}$ e) $\frac{3\pi}{2}$ f) $\frac{5\pi}{18}$ g) $\frac{2\pi}{15}$ 2. a) 120° b) 225° c) 330° d) 72°

3. a) IV b) III c) II 4. a) $\frac{1}{2}$ b) $-\frac{\sqrt{3}}{3}$ c) $\frac{\sqrt{2}}{2}$ d) $-\frac{\sqrt{3}}{3}$ e) -2 f) $\sqrt{2}$

VII. 1. a) amp = 3 b) amp = 2 c) no amp d) amp = $\frac{3}{4}$ e) amp = $\frac{\pi}{2}$ f) amp = $\frac{2}{3}$
(scale factor = 9) (scale factor = $\frac{2}{3}$)

per = π per = 4π per = $\frac{\pi}{3}$ per = 24 per = $\frac{1}{2}$ per = 2
freq = 2 freq = $\frac{1}{2}$ freq = 3 freq = $\frac{\pi}{12}$ freq = 4π freq = $\frac{\pi}{2}$



3. a) $y = -3.5 \cos 2x$

b) $y = -2 \cos 2\left(x + \frac{\pi}{12}\right)$

c) $y = 3 \sin \pi x$

d) $y = 2 \sec x$

VIII. solutions will vary

IX. 1. $\left\{1.25, \frac{3\pi}{4}, 4.39, \frac{7\pi}{4}\right\}$ 2. $\left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}\right\}$ 3. $\left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

4. $\left\{-\pi, -\frac{2\pi}{3}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$ 5. $\left\{\frac{1}{2}, \frac{5}{2}, \frac{7}{2}, \frac{11}{2}, \frac{13}{2}, \frac{17}{2}, \frac{19}{2}\right\}$ 6. $\left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \pi, \frac{3\pi}{2}\right\}$

7. $\left\{\frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \frac{13}{4}, \frac{17}{4}, \frac{21}{4}, \frac{25}{4}\right\}$ 8. $\left\{\frac{2}{3}, \frac{10}{3}, \frac{26}{3}, \frac{34}{3}\right\}$

X. 1.a) $x=5\sqrt{2}, y=5\sqrt{2}$ b) $x=7, y=7\sqrt{3}$ c) $x=8\sqrt{3}, y=4\sqrt{3}$ 2. 1764π 3. $A = \frac{s^2\sqrt{3}}{4}, 108\sqrt{3}$

4. 88 in^2 5. 96π 6. 412 m^2 7. 157.5 cm^3

XI. 1. 245 2. Repeating numbers by 67,416,960 3. a. 479,001,600 b. 1320 c. 220

4. a. $x^3 + 3x^2 + 3x + 1$ b. $x^3 - 6x^2 + 12x - 8$ c. $27x^3 - 54x^2 + 36x - 8$ d. $x^6 - 12x^4 + 48x^2 - 64$

e. $x^4 - 8x^3 + 24x^2 - 32x + 16$ f. $16x^4 + 96x^3 + 216x^2 + 216x + 81$ g. $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

The following information will be necessary for a successful year:

- **Supplies:** Notebook→may be spiral, composition book, or binder with loose leaf; Folder or binder for returned classwork, quizzes, & tests; Graph paper; Straightedge; Pencil/pen; Textbook→should be covered.
- **Calculator:** Graphing calculators are required for the AP exam. Graphing calculators will be made available in class, but the students' ability to use it is greatly improved by practice at home. The model used in class will be the TI-84 Plus CE, made by Texas Instruments. The TI-83 is similar. The TI-Nspire CAS is an advanced calculator that is allowed by the College Board and may also be used.
- **Grading:** Your quarterly average is based on total points earned divided by total points possible. This one semester course ends in a final exam in January. Each quarter counts as 40% of your final grade, and the final exam counts as 20% of the final grade. Continuation on to the 2nd half of the course is contingent upon a passing average in the first semester.
- **Tests & Quizzes:** Tests, which cover an entire unit, will be announced ahead of time. Quizzes, which cover smaller sections of a unit, may be announced or unannounced. Anyone AWOL on the day of an assessment will receive a zero. ***There will be no curves, shifts, or retests.***
- **Homework:** Will be given almost daily, reviewed the next day, and collected at a later date. Not every assignment will be collected. When collected, credit will be assigned for completeness, and a random selection of problems will be graded for accuracy. All work must be shown for all problems. All assignments must be labeled clearly and legible, otherwise no credit will be given. When collected, **ALL** homework is due at the time of collection, and no later. ***Handing in homework when it is collected is optional. When not handed in, the points will not count against your grade.***
- **Attendance:** When absent for ANY reason, **YOU** are responsible for getting missed assignments and notes. Tests & quizzes will be made up **immediately**. Anyone AWOL on the day of an assessment will receive a zero. Being on time is important. This is your warning. Any tardiness without written permission from a staff member of Columbia High School may result in a detention assignment.
- **Being Prepared:** ESSENTIAL!! Bring your notebook, textbook, and pen or pencil to EVERY SINGLE class. Also, use of the restroom should be done outside of class time. *Habitual* problems of this nature will become disciplinary issues.
- **Cell Phones:** Use of a cell phone without permission will not be tolerated. Students are not to use or display any communication device, including cell phones, during assessments of any type, such as writing assignments, quizzes, tests, Regents exams, etc. Students observed using any prohibited communication device during these assessments will receive a zero.
- **PLAGIARISM/CHEATING POLICY:** **ALL WORK SHOULD BE YOUR OWN, UNLESS OTHER INSTRUCTIONS ARE GIVEN.** Any student who commits plagiarism (the use of another's words or ideas, or including phrases within a writing assignment as his or her own without proper citation, or copying solutions from a teacher edition or the Internet) will IMMEDIATELY receive a ZERO for that assignment. The same consequence holds true for any instance of cheating, including but not limited to sharing homework assignment answers through various means of communication. Please be aware that a student who has completed the work and shared it with other students will receive the same consequence.
- **Drop Date:** From the Columbia High School Program of Studies:
"No Dropping of 20 week (1 semester) courses after five weeks"
- **Extra Help:** I am available after school almost every day. Check the math office or make an appointment.

Student _____ Signature _____

Parent/Guardian _____ Signature _____

Date _____