Advanced Placement Calculus BC Summer Practice
Mrs. Van Oort

The AP Calculus Course is a challenging, fast paced effort to prepare students for the AP Exam in May and also to provide a strong experience in preparation for any college calculus courses that may be taken after graduation. As such, the foundation upon which this course stands must be firmly in place. This practice is intended to assure that the mathematics covered in the past four years has been retained at a sufficient level so as to not hinder the development of new concepts with the pondering over fundamental computations.

This practice, with the exception of Section XI, is intended for the student to be able to complete independently, without the need to look up anything in either a text book, or on the internet. It should be noted, that anything the student does need to look up would indicate a need for study in that topic area. All of the material in this practice shall be needed at quick recall at some point during the calculus class. A need to look things up will hinder the student's process of understanding the concepts that will be addressed in this course. Therefore, one of the intentions of this practice is to reveal to the student topic areas that may require some independent remediation, hopefully before starting calculus. Section XI is, however, new material. This will be discussed in the first days of school, and students would benefit from preparation.

This practice is divided into two parts. The first is only informational; each section addresses some fundamental ideas and skills required for that topic. The second section consists of the problems that are to be solved. A reasonable amount of work should be shown for each question. With rare exceptions, an answer standing alone would not be accepted on a graded assessment. When in doubt, show some sort of work or description of your thought process. Answers are provided at the end of the packet, for self–assessment.

If you lose this practice, it will be posted on the Columbia High School Math Department website. Upon the opening of school in September, questions may be asked over the course of the first few class days, during the homework review portion of class. In addition, after school help will be available upon request during that time. ***On Tuesday, September 10, 2024, there will be a quiz in class based on this material.***

Section A – Information

I. Polynomials & Equations

ⓘIDEAS

- ☑ multiplying polynomials
- ☑ factoring trinomials
- ☑ factoring difference of 2 squares
- ☑ greatest common factors
- ☑ factoring sum/difference of cubes
- ☑ factor by grouping
- ☑ solving quadratics by factoring
- ☑ solving quadratics by quadratic formula
- ☑ solving quadratics by completing the square
- ☑ solve higher order polynomial equations algebraically
- ☑ solve radical equation
- ☑ use fractional exponents
- ☑ simplify complex fractions
- ☑ solve equations containing rational expressions
- ☑ calculate rational exponents without a calculator

CALCULATOR SKILLS

- ☑ graph polynomial equations
- ☑ find extrema
- ☑ find any y-value given an x-value, **not** using the table
- ☑ solve equations graphically
- ☑ use appropriate zoom and adjust window
- ☑ find roots
- ☑ format and use the table

II. Exponentials & Logarithms

ⓘIDEAS

- ☑sketch graph of exponential function
- ☑ determine domain and range of logarithmic function
- ☑ solve exponential equations
- ☑ determine domain and range of exponential function
- ☑ use properties of logarithms to manipulate log expressions
- ☑ solving logarithmic equations
- ☑ sketch graph of logarithmic function
- ☑ change the base of a logarithmic expression
- ☑ switch between exponential form and logarithmic form

CALCULATOR SKILLS

- ☑ graph exponential equations
- ☑ use base e to calculate
- ☑graph logarithmic equations
- ☑ calculate logarithmic values
- ☑ solve equations graphically

III. Graphing Polynomials and Rational Functions

ⓘIDEAS

- ☑ find slope of line given two points
- ☑ recognize intervals of increase or decrease on a graph
- ☑ find vertical and horizontal asymptotes algebraically
- ☑ determine algebraically and graphically if a function is even or odd
- ☑ write equation of line in slope intercept form
- ☑ determine turning points or vertices of graphs
- ☑ find x and y intercepts algebraically
- ☑ use long division to find factors and roots
- ☑ write equation of line in point intercept form
- ☑ determine end behavior of a graph from the equation
- ☑ use synthetic division to find factors and roots

CALCULATOR SKILLS

- ☑ graph polynomial equations
- ☑ find any y-value given an x-value, **not** using the table
- ☑ find extrema
- ☑ graph polynomial equations
- ☑ use appropriate zoom and adjust window
- ☑ find roots
- ☑ format and use the table

IV. Transformations of Graphs

ⓘIDEAS

- ☑ transformation rules for translation, reflection, scaling
- ☑ complete the square on quadratic to find vertex
- ☑ parent graphs of the following:

$f(x) = x^2$

$f(x) = \sqrt{x}$

$f(x) = b^x$

$f(x) = \frac{1}{x}$

$f(x) = \log x$

$f(x) = \ln x$

$f(x) = |x|$

CALCULATOR SKILLS

- ☑ graph equations
- ☑ use multiple graphs simultaneously

V. Conic Sections

ⓘIDEAS

- ☑ recognize standard form of equation for parabola, circle, ellipse, hyperbola
- ☑ sketch a graph using information form equation to determine features such as vertices, foci, axis of symmetry, major axis, minor axis, transverse axis, conjugate axis
- ☑ complete the square to put equation in standard form
- ☑ write an equation based on the graph

CALCULATOR SKILLS

- ☑ graph using multiple equations
- ☑ solve equations graphically

VI. Trigonometry Basics

ⓘIDEAS

☑ convert degrees←→radians

☑ know common angles in radians $\left(90° = \frac{\pi}{2}, 45° = \frac{\pi}{4}, 30° = \frac{\pi}{6}, \text{etc.}\right)$

☑ signs of trig functions in the different quadrants (ASTC)

☑ 21 club values→**NO CALC!!!**

☑ reference angles (degrees/radians)

☑ reciprocal functions

	0° (0)	30°(π/6)	45°(π/4)	60°(π/3)	90°(π/2)	180°(π)	270°(3π/2)
sinθ	0	1/2	$\sqrt{2}/2$	$\sqrt{3}$	1	0	–1
cosθ	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0	–1	0
tanθ	0	$\sqrt{3}/3$	1	$\sqrt{3}$	undef	0	undef

CALCULATOR SKILLS

☑ use appropriate mode

☑ calculate trig values

☑ convert degrees←→radians in a rational result (with π)

☑ appropriate use of inverse functions

VII. Trigonometry Graphs

ⓘIDEAS

☑ amplitude

☑ transformations of basic graphs

☑ graphs of all 6 trig functions

☑ frequency (including π)

☑ work in radians

☑ period (including rational)

☑ write equations based on graphs

CALCULATOR SKILLS

☑ graph trig functions

☑ work with multiple equations simultaneously

☑ work in radians

☑ use appropriate zoom and adjust window

☑ format and use the table

VIII. Trigonometric Identities

ⓘIDEAS

☑ Pythagorean identities

☑ quotient identities

☑ double angle identities

☑ reciprocal identities

☑ be able to substitute and manipulate trig identities to simplify expressions

IX. Trigonometric Equations

ⓘIDEAS

☑ solve algebraically using identities ☑ work with frequencies other than 1 ☑ work in various intervals

CALCULATOR SKILLS

☑ solve graphically ☑ work in radians ☑ format and use the table
☑ use appropriate zoom and adjust window

X. Geometry

ⓘIDEAS

☑ special right triangles → 30–60–90 and 45–45–90
☑ area formulae for: triangle, rectangle, square, parallelogram, trapezoid, circle
☑ volume formulae for: rectangular prism, pyramid, cylinder, cone, sphere

XI. Counting Principles and Binomial Expansion

This is a new topic. Do your best to understand it. Khan Academy has some videos that address it. We will discuss this when you return to school, under the assumption that you have attempted to understand.

ⓘIDEAS

☑ counting principal→how many ways to do something.

Multiply the number of options for each choice:

A woman has 8 blouses, 5 skirts and 6 scarves. How many outfits can be assembled from these?

__8__ X __5__ X __6__ = 240 outfits
blouses skirts scarves

A license plate consists of consists of 3 letter and 3 numbers. How many different plates possible?

__26__ X __26__ X __26__ X __10__ X __10__ X __10__ = 26^3 X 10^3 = 17,576,000

☑ ordered arrangements→putting a whole group in order

Putting 3 people in order in a line: ABC, ACB, BAC, BCA, CAB, CBA
Counting principle: ___3___ X ___2___ X ___1___ = 6
available # available # available
for 1st spot for 2nd spot for 3rd spot
Uses a function called *factorial*, whose symbol is after the # → this is 3!
4! = 4X3X2X1 5! = 5X4X3X2X1 $n!$ = nX (n–1) X (n–2) X…X3X2X1
***Fact**: 0! = 1 …weird because of definitions

☑ ordered arrangements➔putting part of a group in order

Taking 5 people, selecting 3 to stand in order in a line

People ABCDE➔in line ABC, BCD, CDE, AEB, DCA, etc.

Counting principle: ___5___ X ___4___ X ___3___ = 60

available for 1st spot | # available for 2nd spot | # available for 3rd spot

This is a function called a *permutation*, whose symbol is ${}_nP_r$, where n = total number of objects to choose from, and r = number of objects chosen. This one is ${}_5P_3$.

***Fact**: choosing all the items is the same as the factorial➔${}_5P_5 = 5!$

☑ arrangements where order doesn't matter➔selecting committees or teams

Taking 5 people, selecting 3 to represent the group (doesn't matter who was chosen 1st)

People ABCDE➔ ABC is same as BAC, CBA, BCA, etc.

Counting principle hard to apply here

This is a function called a *combination*, whose symbol is ${}_nC_r$, where n = total number of objects to choose from, and r = number of objects chosen. This one is ${}_5C_3$.

Combinations can be calculated using permutations and factorials: ${}_nC_r = \frac{{}_nP_r}{(n-r)!}$

$${}_5C_3 = \frac{{}_5P_3}{(5-3)!} = \frac{5 \bullet 4 \bullet 3}{2!} = \frac{5 \bullet 4 \bullet 3}{2 \bullet 1} = 30$$

***Facts**: There is only one way of choosing none of the items➔ ${}_nC_0 = 1$

There is only one way of choosing all of the items➔ ${}_nC_n = 1$

There are n ways to choose one object ➔ ${}_nC_1 = n$

There are n ways to choose all but one object ➔ ${}_nC_{n-1} = n$

There are pairs of choosing items that are the same➔ ${}_nC_r = {}_nC_{n-r}$

Add up to n

Examples: ${}_5C_3 = {}_5C_2 = 30$; ${}_8C_3 = {}_8C_5 = 56$; ${}_{100}C_1 = {}_{100}C_{99} = 100$

☑ expanding a binomial raised to a power

$(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$

$(x+y)^3 = (x+y)(x+y)(x+y) = x^3 + 3x^2y + 3xy^2 + y^3$ ➔any power higher than 3 is difficult and tedious

Binomial Expansion➔uses known patterns and combinations as a shortcut

$(x+y)^n = {}_nC_0(x)^n(y)^0 + {}_nC_1(x)^{n-1}(y)^1 + {}_nC_2(x)^{n-2}(y)^2 + {}_nC_3(x)^{n-3}(y)^3 + \ldots + {}_nC_{n-1}(x)^1(y)^{n-1} + {}_nC_n(x)^0(y)^n$

$(x+y)^5 = {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^4(y)^1 + {}_5C_2(x)^3(y)^2 + {}_5C_3(x)^2(y)^3 + {}_5C_4(x)^1(y)^4 + {}_5C_5(x)^0(y)^5$

$= 1x^5 + 5x^4y + 30x^3y^2 + 30x^2y^3 + 5xy^4 + 1y^5$

$(x+2)^4 = {}_4C_0(x)^4(2)^0 + {}_4C_1(x)^3(2)^1 + {}_4C_2(x)^2(2)^2 + {}_4C_3(x)^1(2)^3 + {}_4C_4(x)^0(2)^4$

$= 1x^4 + 4x^3(2) + 6x^2(2)^2 + 4x(2)^3 + 1(2)^4$

$= x^4 + 8x^3 + 24x^2 + 32x + 16$

$(x-3)^3 = {}_3C_0(x)^3(-3)^0 + {}_3C_1(x)^2(-3)^1 + {}_3C_2(x)^1(-3)^2 + {}_3C_3(x)^0(-3)^3$

$= 1x^3 + 3x^2(-3) + 3x^1(-3)^2 + 1(-3)^3$

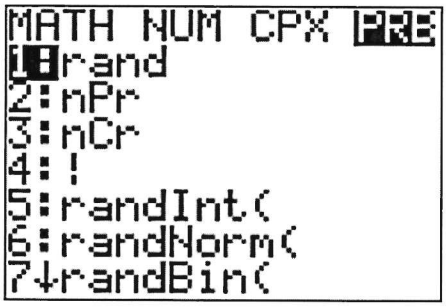
$= x^3 - 9x^2 + 27x - 27$

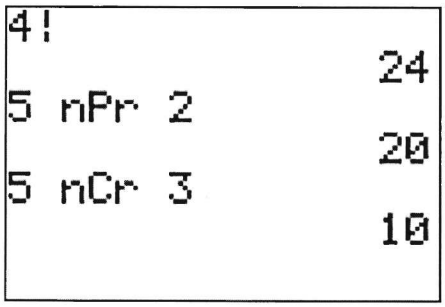
CALCULATOR SKILLS

☑ all 3 functions (factorials, permutations, combinations) can be calculated on the graphing calculator

Older operating systems:
MATH ➔➔➔PROB
Choices 2, 3, and 4

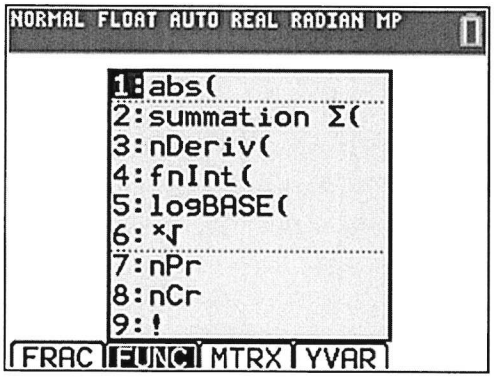
Have to enter numbers in proper order
For each individual function

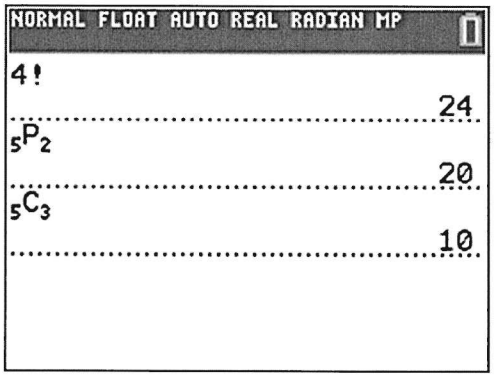




Newer operating systems:
ALPHA➔WINDOW
Choices 7, 8, and 9

Enter factorial number before
For Permutation and combinations
fill numbers into blanks





XII. Parametric Equations

ⓘIDEAS

☑ sketch the graph of a curve given by a set of parametric equations
☑ eliminate the parameter in a set of parametric equations

CALCULATOR SKILLS

☑ graph a set of parametric equations

XIII. Polar Coordinates and Polar Equations

ⓘ IDEAS

☑ polar coordinates (r, θ) are related to the rectangular coordinates (x, y) as follows:

$x = r\cos\theta$ $\quad y = r\sin\theta$ $\quad \tan\theta = \frac{y}{x}$ $\quad r^2 = x^2 + y^2$

☑ rewrite rectangular coordinates in polar form and vice versa
☑ find the zeros and maximum r-values
☑ special polar graphs: limaçons, rose curves, circles, lemniscates
☑ find the points of intersection of two polar graphs
☑ rewrite rectangular equations in polar form and vice versa
☑ sketch the graph of an equation in polar form
☑ find the interval for θ over which the graph is traced *only once*

CALCULATOR SKILLS

☑ convert rectangular←→polar
☑ graph polar equations
☑ format and use the table
☑ work with multiple equations simultaneously
☑ work in radians
☑ use appropriate zoom and adjust window

Section B – Problems

I. Polynomials & Equations

Factor the following polynomials completely:

1. $6x^2 - 13x + 2$
2. $9x^2 - 12x + 4$
3. $5x^3 - x^2 - 5x + 1$
4. $x^3 + 3x^2 - x - 3$
5. $2x^3 + 13x^2 + 15x$
6. $4x^3 - 10x^2 + 6x$
7. $x^4 - 12x^2 + 27$
8. $9 - 9(x+2)^2$
9. $3x^3 - 12x^2 + 6x - 24$
10. $5x^4 - 5y^4$
11. $5x^2 + 22x - 15$

Calculate without a calculator

12. $32^{\frac{3}{5}}$
13. $64^{\frac{-2}{3}}$
14. $(-8)^{\frac{5}{3}}$
15. $\left(\frac{9}{4}\right)^{\frac{-5}{2}}$

Solve the following equations algebraically for x:

16. $6x^2 + x - 12 = 0$
17. $x^3 = x$
18. $x^4 - 3x^2 + 2 = 0$
19. $2x^3 - 6x^2 - 6x + 18 = 0$
20. $\sqrt{2x+7} - x = 2$
21. $\sqrt{2x+6} - \sqrt{x+4} = 1$

22. $(x+1)^{\frac{2}{3}} = 4$

23. $(3-x)^{\frac{4}{3}} + 2 = 18$

24. $8(2x+1)^{-\frac{3}{2}} = 27$

25. $\frac{x}{3} + \frac{3x}{4} = 2$

26. $\frac{2}{x} = \frac{3}{x-2} - 1$

27. $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$

Solve by completing the square:

28. $x^2 + 6x - 5 = 0$

Solve by the quadratic formula:

29. $2x^2 + 3x - 1 = 0$

II. Exponentials & Logarithms

1. Write the expression $\log_2 \sqrt{\frac{x^2}{y}}$ in terms of $\log_2 x$ and $\log_2 y$.

2. Write as the logarithm of a single quantity: $\frac{1}{4}\log_b 16 - 2\log_b 5 + \log_b 7$

3. Find the domain of $f(x) = \log(x - 2)$.

4. If $A = A_0 e^{rt}$, find $\ln A$ in simplest form.

5. Solve for x, rounding to the nearest hundredth:

   a) $4 + e^x = 6.72$ b) $y = \log_3 7$ c) $\log_5(x+3) - \log_5 x = \log_5 4$

   d) $\frac{1}{2}\ln x = 4\ln 2 - \ln 4$ e) $2^{3x} = 7$ f) $e^x = 7$

III. Graphing Polynomials and Rational Functions

1. Determine the left-hand and right-hand behavior of the graphs of the following functions:

a) $f(x) = -x^5 + 2x^2 - 1$ b) $f(x) = 2x^3 + 5x^2 - 8$ c) $f(x) = -7x^4 + \frac{1}{2}x^3 - 3$ d) $f(x) = 4x^6 + 8x^5 - 2x^2 + 4$

2. Determine all zeros, asymptotes, and intercepts of the graph of

a) $f(x) = \dfrac{6x+3}{x^2-1}$ b) $f(x) = \dfrac{x^2-4}{x^2-6x+5}$

3. Divide using long division $(2x^5 - 3x^3 + 5x^2 - 1) \div (2x + 3)$

4. Divide using synthetic division $\dfrac{2x^4 - 5x^3 + 7x^2 - 3x + 1}{x - 3}$

5. a) Determine if $x - 3$ a factor of $x^3 - x^2 - 14x + 1$. b) Determine if $x - 1$ a factor of $2x^3 + x^2 - 5x + 2$

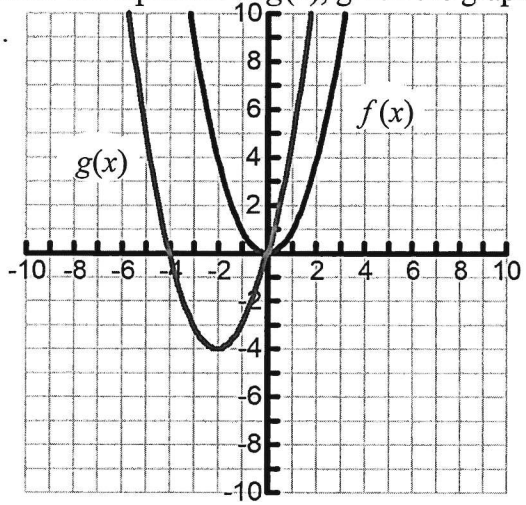
6. Determine if the following functions are even, odd, or neither…justify your answer.

a) $f(x) = 3x^4 - 6x^2$ b) $f(x) = -x^6 - 2x^4$ c) $f(x) = x^{\frac{2}{3}}$ d) $f(x) = x\sqrt{x-3}$

IV. Transformations of Graphs

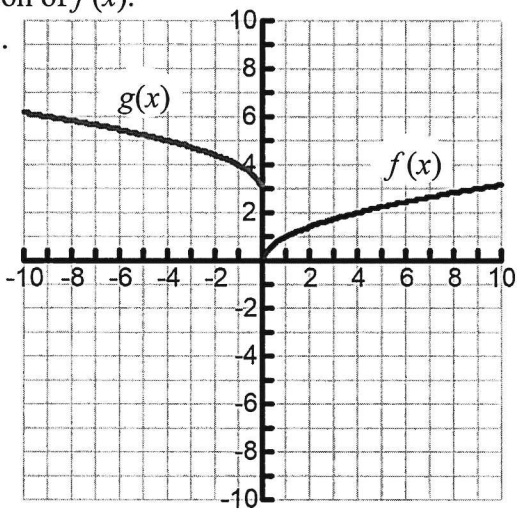
Write the equation of $g(x)$, given the graph and equation of $f(x)$.

1.



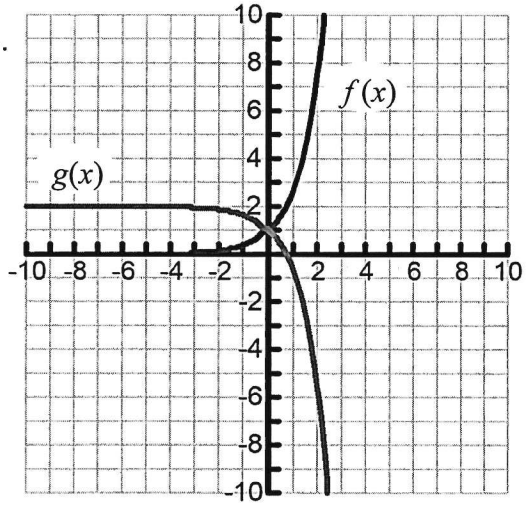
$f(x) = x^2$

2.



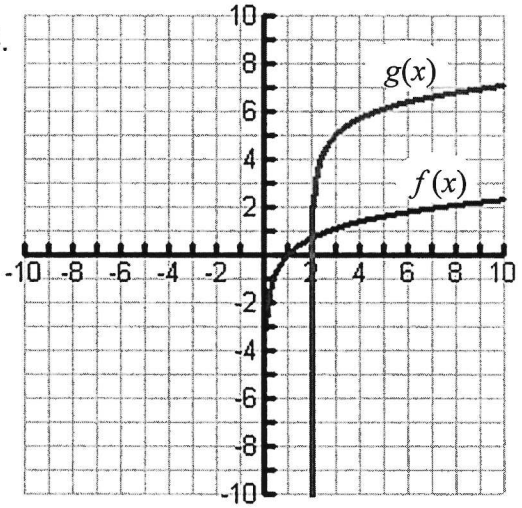
$f(x) = \sqrt{x}$

3.



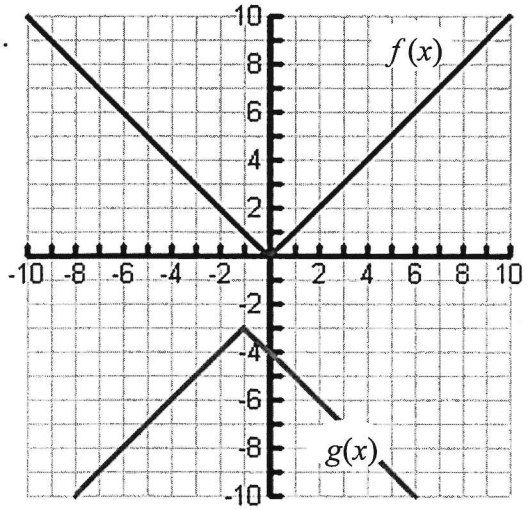
$f(x) = e^x$

4.



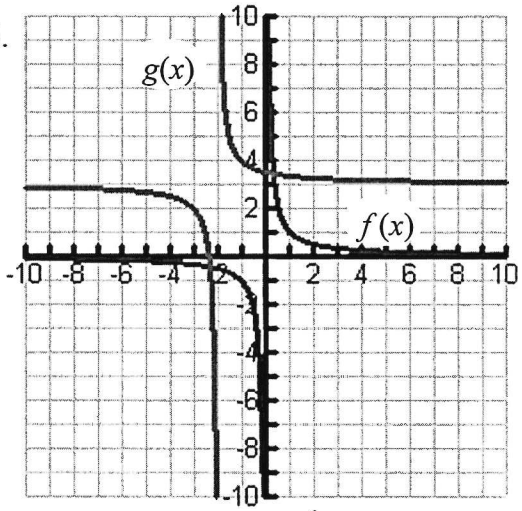
$f(x) = \ln x$

5.



$f(x) = |x|$

6.

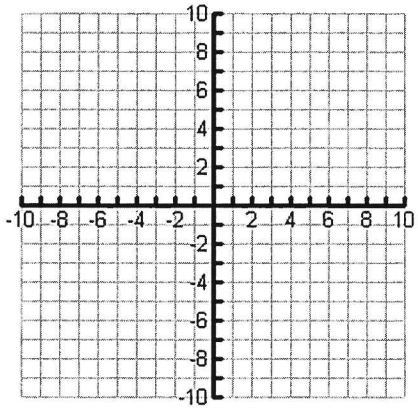


$f(x) = \frac{1}{x}$

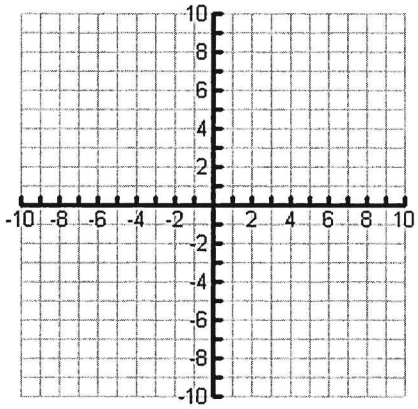
V. Conic Sections

1. Write the equation in the standard form for its type and sketch a graph:

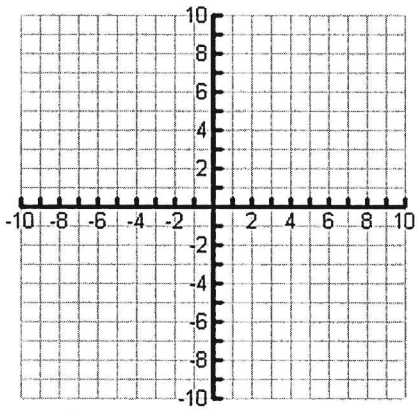
a) $y^2 - 10y + 12x + 37 = 0$



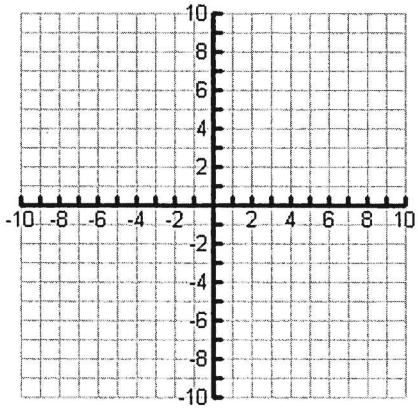
b) $9x^2 - 36x + y^2 + 14y = -76$



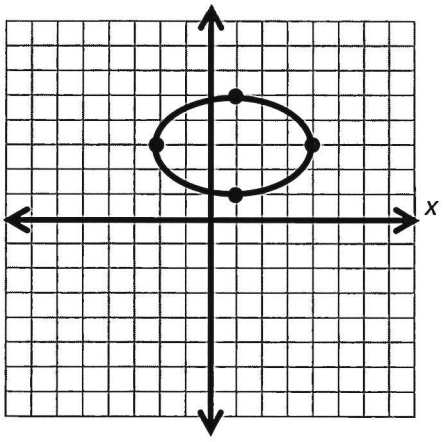
c) $3x^2 + 6x - 2y^2 - 12y = 21$



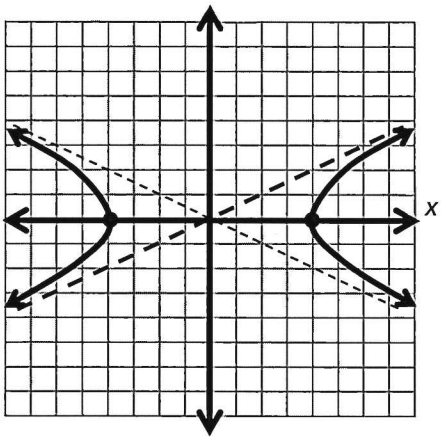
d) $x^2 + y^2 - 4x + 8y + 14 = 0$



2. Write the standard form of the equation for the ellipse shown here:



3. Write the standard form of the equation for the hyperbola shown here:



VI. Trigonometry Basics

1. Convert the degree measure to radian measure, in terms of π:

a) $45°$ b) $90°$ c) $30°$ d) $60°$ e) $270°$ f) $50°$ g) $24°$

2. Convert the radian measure to degree measure:

a) $\frac{2\pi}{3}$ b) $\frac{5\pi}{4}$ c) $\frac{11\pi}{6}$ d) $\frac{2\pi}{5}$

3. Determine the quadrant in which the terminal side of θ lies:

a) If $\cos\theta > 0$ and $\sin\theta < 0$ b) if $\sec\theta < 0$ and $\cot\theta > 0$ c) if $\csc\theta > 0$ and $\tan\theta < 0$

4. Determine the **EXACT** function value, in simplest form, without a calculator:

a) $\cos\frac{\pi}{3}$ b) $\tan\frac{5\pi}{6}$ c) $\sin\frac{\pi}{4}$ d) $\cot\frac{2\pi}{3}$ e) $\csc\frac{11\pi}{6}$ f) $\sec\frac{7\pi}{4}$

VII. Trigonometric Graphs

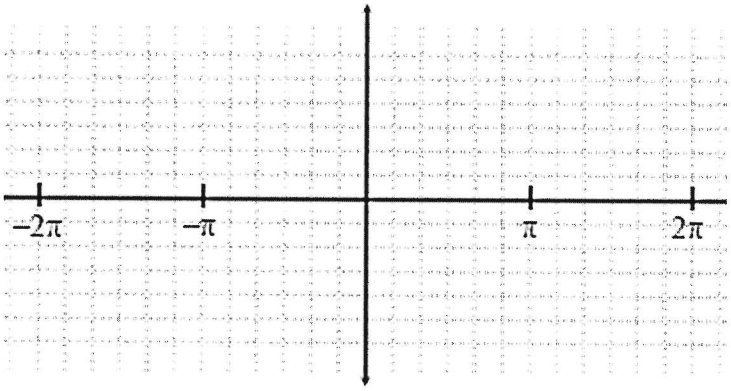
1. Find the amplitude, period and frequency for each equation:

a) $y = 3 \sin 2x$ b) $f(x) = -2 \cos \frac{1}{2}x$ c) $y = 9 \tan 3\theta$

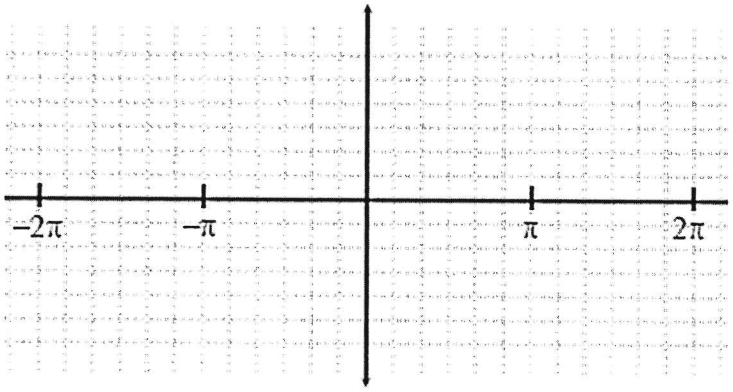
d) $s(t) = -\frac{3}{4} \sin \frac{\pi}{12} t$ e) $f(\theta) = \frac{\pi}{2} \cos 4\pi\theta$ f) $y = \frac{2}{3} \tan \frac{\pi x}{2}$

2. Sketch a graph by hand, then verify with a graphing calculator

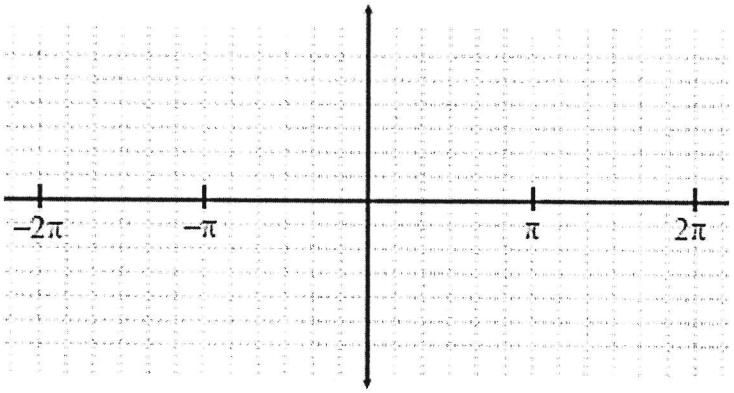
a) $y = 2 \sin 3\left(x + \frac{\pi}{3}\right) - 1$ on $(-2\pi, 2\pi)$



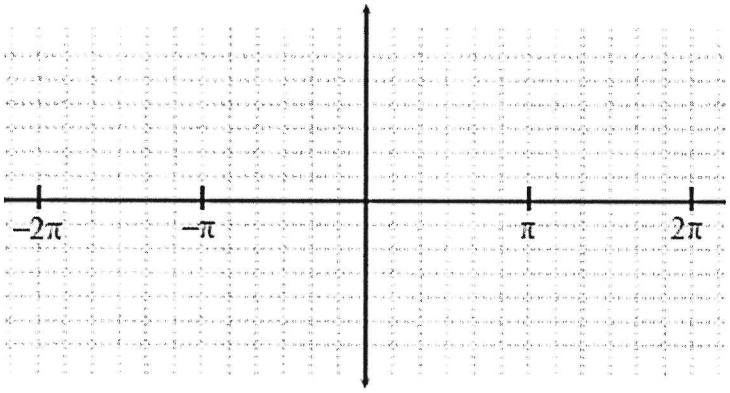
b) $y = -3 \cos \frac{1}{2}\left(x - \frac{\pi}{6}\right) + 1$ on $(-2\pi, 2\pi)$



c) $y = \tan \frac{x}{2}$ on $(-2\pi, 2\pi)$

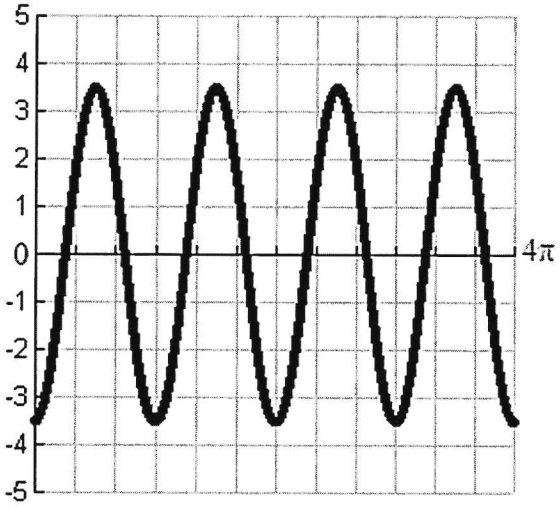


d) $y = 2\csc x$ on $(-2\pi, 2\pi)$

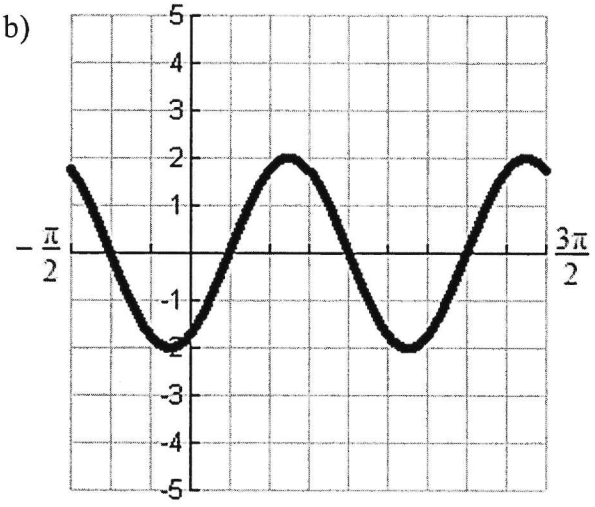


3. Write the equation for the given graph, using the smallest horizontal shift possible.

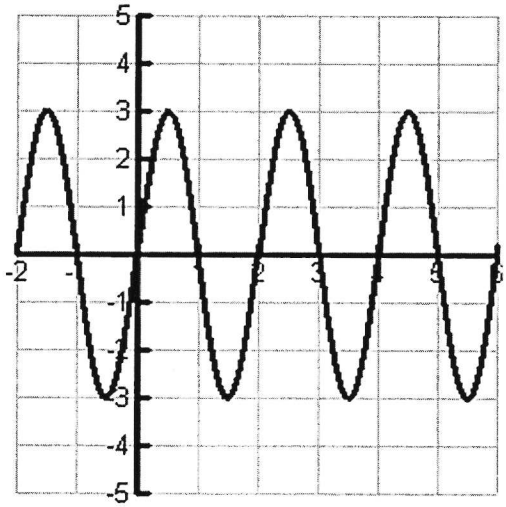
a)



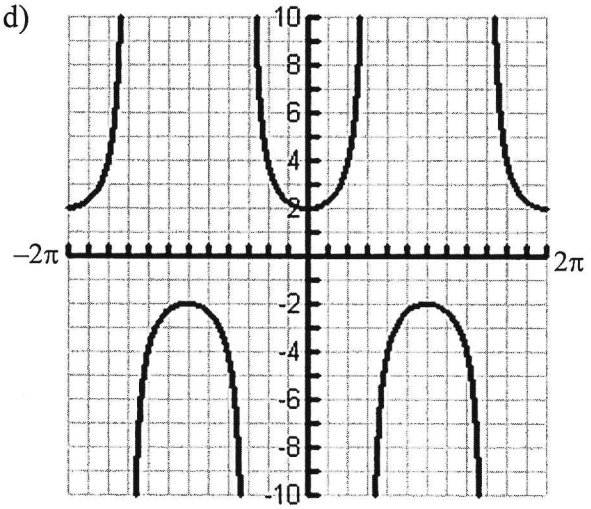
b)



c)



d)



VIII. Trigonometric Identities

Verfiy the following identities:

1. $\dfrac{2\cos x}{\sin 2x} = \csc x$

2. $\dfrac{\cos^2 x}{1+\sin x} = 1-\sin x$

3. $\sec^2 x(\cos 2x) = 2-\sec^2 x$

4. $\csc^2 x(\sec^2 x - 1) = \sec^2 x$

5. $1-\cos 2x + 2\cos^2 x = 2$

IX. Trigonometric Equations

Algebraically, find exact values of all solutions in the interval stated for each. Round to the nearest hundredth if exact value not possible.

1. $\sec^2 x - 2\tan x = 4$ on $[0,2\pi)$

2. $4\sin x \cos x = \sqrt{2}$ on $[0,2\pi)$

3. $6\cos^2(2x) - 3 = 0$ on $[0,2\pi)$

4. $\sin 3x = 0$ on $[-\pi,\pi]$

5. $4\cos^2\left(\frac{\pi}{3}x\right) - 3 = 0$ on $[0,3\pi]$

6. $(\sin 2x + \cos 2x)^2 = 1$ on $[0,2\pi)$

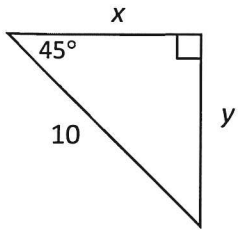
7. $3\tan(\pi x) - 3 = 0$ on $[0,2\pi)$

8. $2\csc^3\left(\frac{\pi}{4}x\right) - 10 = 6$ on $[0,4\pi]$

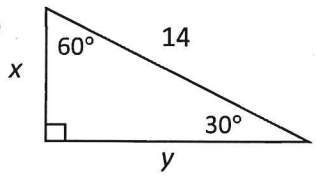
X. Geometry

1. Find x and y in simplest radical form:

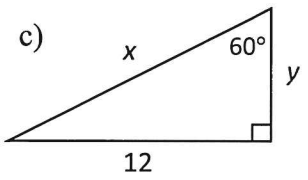
a)



b)



c)



2. The circumference of a circle is 84π. Find its area.

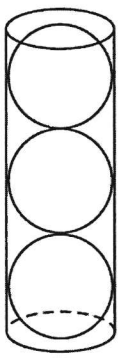
3. Derive the formula for the area of an equilateral triangle whose side is s. Find the exact area of an equilateral triangle whose side is $12\sqrt{3}$ ft.

4. An isosceles trapezoid has legs which measure 10 inches, and bases that measure 5 and 17 inches, respectively. Find the area of this trapezoid.

5. A cone has a radius of 3 and a volume of 12π. A similar cone has a height of 8. Find the volume of the second cone, in terms of π.

6. A rectangular prism has a length, width, height ratio of $\frac{7}{2} : 4 : 5$. If it has a volume of 560 m^3, find its surface area.

7. Three tennis balls are packed so that they touch each end and the sides of their can exactly. If the volume of the can is 708.6561235 cm^3, find the volume of a single tennis ball, to the nearest tenth of a cm^3.



XI. Counting Principles and Binomial Expansion

1. Megan decides to go out to eat. The menu at the restaurant has four appetizers, three soups, seven entrées and five desserts. If Megan decides to order an appetizer *or* a soup, one entrée, and one dessert, how many different choices can she make?

2. One state issues license plates with 3 letters and 4 numbers, allowing only numbers to repeat. Another state uses the same 3 letter, 4 number set up, but only allows repetition of the letters. Which state will have more available plates, and by how many?

3. 12 people run in a race. Find the following:

   a. The number of ways they can finish the race.

   b. The number of arrangements of a 1^{st}, 2^{nd} , and 3^{rd} place finish.

   c. The number of ways you can randomly select any 3 participants for a drug test.

4. Expand the following, using the binomial expansion;

   a. $(x + 1)^3$

   b. $(3x - 2)^3$

   c. $(x^2 - 4)^3$

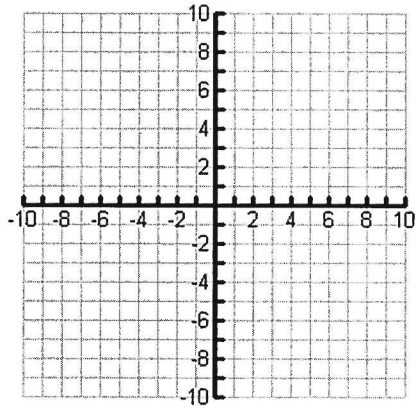
   d. $(x - 2)^4$

   e. $(2x + 3)^4$

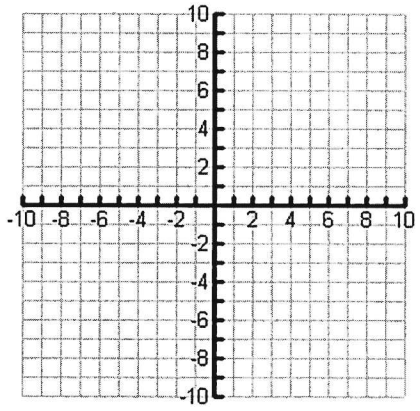
XII. Parametric Equations

Sketch the curve represented by the parametric equations (indicate the direction of the curve). Then eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve.

1. $x = t + 1$
   $y = t^2$



2. $x = 4 + 2\cos\theta$
   $y = -1 + \sin\theta$



XIII. Polar Coordinates and Polar Equations

1. Given the polar coordinates, find the corresponding rectangular coordinates of the point.

   a) $\left(4, \frac{-\pi}{3}\right)$ b) $\left(-1, \frac{-3\pi}{4}\right)$ c) $\left(-2, \frac{7\pi}{6}\right)$ d) $\left(2, \frac{5\pi}{3}\right)$

2. Given the rectangular coordinates, find two sets of polar coordinates of the point for $0 \le \theta \le 2\pi$.

   a) (4, 6) b) (–3, 4) c) $\left(-\sqrt{3}, -\sqrt{3}\right)$ d) (3, –1)

3. Convert the rectangular equation to polar form.

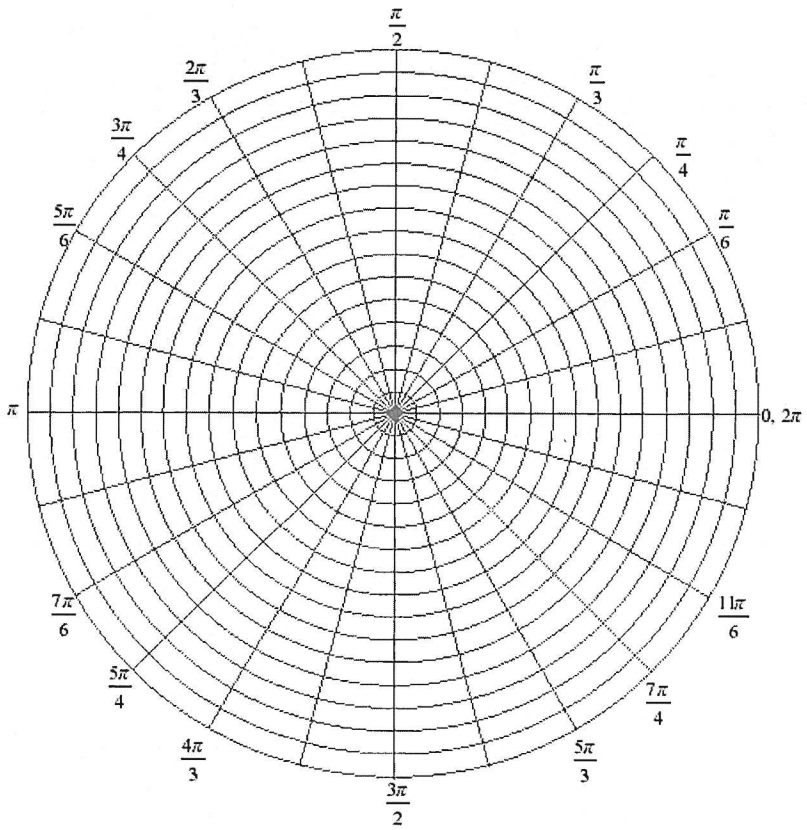
a) $x^2 + y^2 = 9$ b) $3x - y + 2 = 0$ c) $y^2 - 8x - 16 = 0$

4. Convert the polar equation to rectangular form.

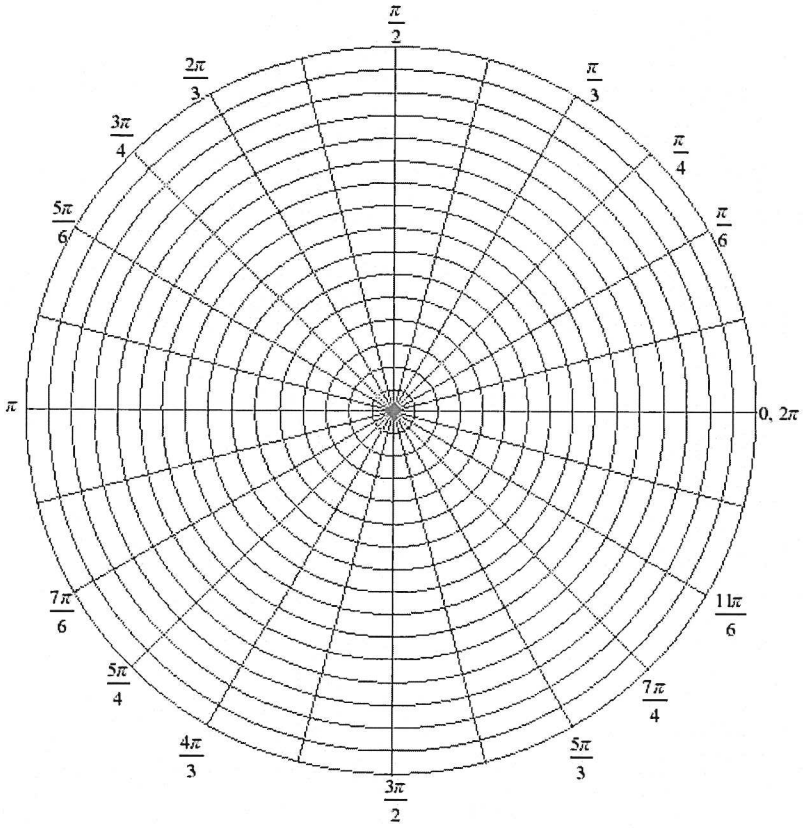
a) $r = 4\sin\theta$ b) $\theta = \frac{\pi}{6}$ c) $r = \frac{6}{2-3\sin\theta}$

5. Sketch the graph of the polar equation (without the use of any technology).

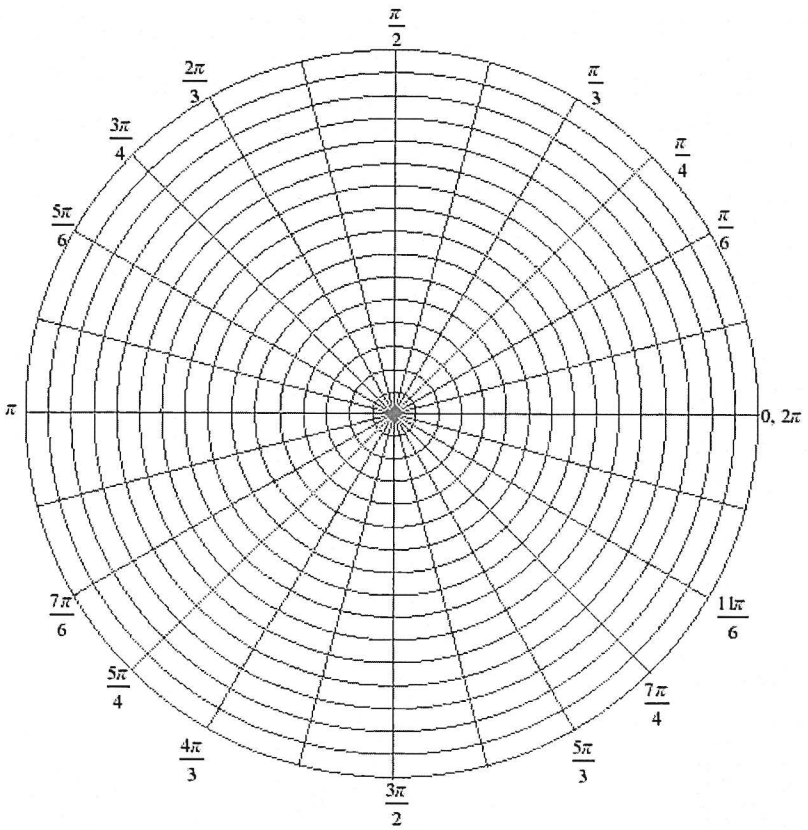
a) $r = 2\cos 3\theta$



b) $r = 2 + 4\sin\theta$



c) $r = 3(1 - \cos\theta)$



Advanced Placement Calculus BC Summer Assignment Answers

I. 1. $(6x-1)(x-2)$ 2. $(3x-2)(3x-2)$ 3. $(5x-1)(x+1)(x-1)$ 4. $(x+3)(x+1)(x-1)$

5. $x(2x+3)(x+5)$ 6. $2x(2x-3)(x-1)$ 7. $(x+3)(x-3)(x+\sqrt{3})(x-\sqrt{3})$ 8. $-9(x+3)(x+1)$

9. $3(x^2+2)(x-4)$ 10. $5(x^2+y^2)(x+y)(x-y)$ 11. $(5x-3)(x+5)$ 12. 8 13. $\frac{1}{16}$ 14. -32 15. $\frac{32}{243}$

16. $\left\{-\frac{3}{2},\frac{4}{3}\right\}$ 17. $\{-1,0,1\}$ 18. $\left\{-\sqrt{2},-1,1,\sqrt{2}\right\}$ 19. $\left\{-\sqrt{3},,3,\sqrt{3}\right\}$ 20. $\{1\}$ 21. $\{5\}$ 22. $\{-9,7\}$

23. $\{-5, 11\}$ 24. $\left\{-\frac{5}{18}\right\}$ 25. $\left\{\frac{24}{13}\right\}$ 26. $\{-1,4\}$ 27. $\{\ \}$ 28. $\left\{-3\pm\sqrt{14}\right\}$ 29. $\left\{\frac{-3\pm\sqrt{17}}{4}\right\}$

II. 1. $\log_2 x-\frac{1}{2}\log_2 y$ 2. $\log_b \frac{14}{25}$ 3. $\{x|x>2\}$ or $(2,\infty)$ 4. $\ln A_0+rt$

5. a) 1.00 b) 1.77 c) 1.00 d) 16.00 e) 0.94 f) 1.95

III. 1. a) rises left, falls right b) falls left, rises right c) falls left, falls right d) rises left, rises right

2. a) zeros $x=-½$; vertical asymptotes $x=\pm1$; horizontal asymptote $y=0$; y–int $y=-3$
   b) zeros $x=\pm2$; vertical asymptotes $x=1,5$; horizontal asymptote $y=1$; y–int $y=-4/5$

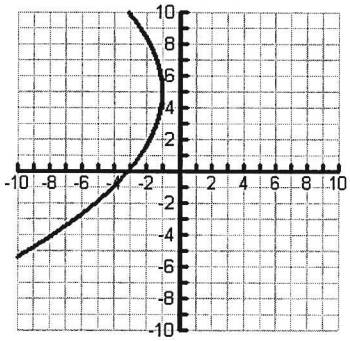
3. $x^4-\frac{3}{2}x^3+\frac{3}{4}x^2+\frac{11}{8}x-\frac{33}{16}+\frac{83}{16(2x+3)}$ 4. $2x^3+x^2+10x+27+\frac{82}{x-3}$ 5.a) no b) yes

6.a) even b) even c) even d) neither

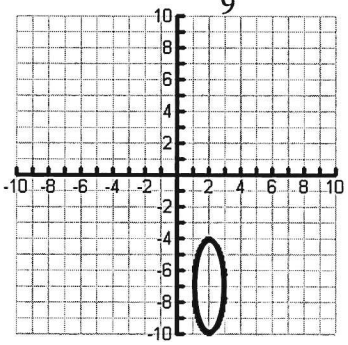
IV. 1. $g(x)=(x+2)^2-4$ 2. $g(x)=\sqrt{-x}+3$ 3. $g(x)=-e^x+2$ 4. $g(x)=\ln(x-2)+5$

5. $g(x)=-|x+1|-3$ 6. $g(x)=\frac{1}{x+2}+3$

V. 1. a) $(y-5)^2=-12(x+1)$



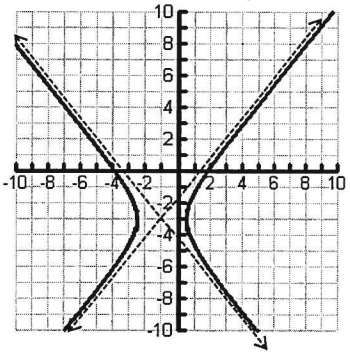
b) $(x-2)^2+\frac{(y+7)^2}{9}=1$



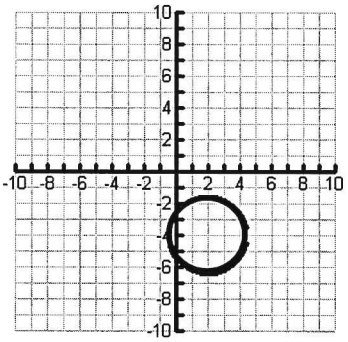
2. $\frac{(x-1)^2}{9}+\frac{(y-3)^2}{4}=1$

3. $\frac{x^2}{16}-\frac{y^2}{4}=1$

c) $\frac{(x+1)^2}{2}-\frac{(y+3)^2}{3}=1$



d) $(x-2)^2+(y+4)^2=6$



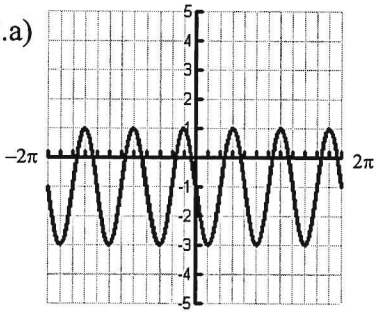
VI. 1.a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{3}$ e) $\frac{3\pi}{2}$ f) $\frac{5\pi}{18}$ g) $\frac{2\pi}{15}$ 2. a) 120° b) 225° c) 330° d) 72°

3. a) IV b) III c) II 4. a) $\frac{1}{2}$ b) $-\frac{\sqrt{3}}{3}$ c) $\frac{\sqrt{2}}{2}$ d) $-\frac{\sqrt{3}}{3}$ e) –2 f) $\sqrt{2}$

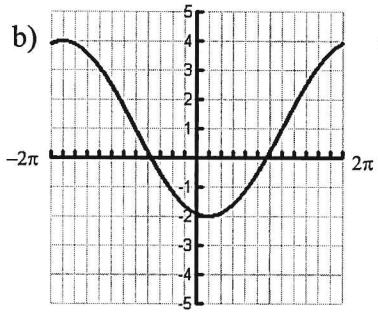
VII. 1. a) amp = 3 b) amp = 2 c) no amp (scale factor of 9) d) amp = 3/4 e) amp = π/2 f) no amp (scale factor of 2/3)

per = π per = 4π per = π/3 per = 24 per = ½ per = 2
freq = 2 freq = ½ freq = 3 freq = π/12 freq = 4π freq = π/2

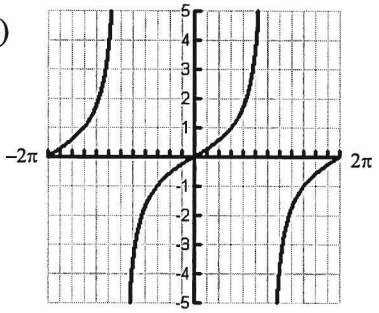
2.a)



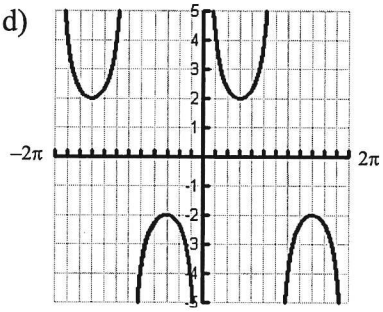
b)



c)



d)



3. a) $y = -3.5 \cos 2x$

b) $y = -2\cos 2\left(x + \frac{\pi}{12}\right)$

c) $y = 3 \sin \pi x$

d) $y = 2 \sec x$

VIII. solutions will vary

IX. 1. $\left\{1.25, \frac{3\pi}{4}, 4.39, \frac{7\pi}{4}\right\}$ 2. $\left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}\right\}$ 3. $\left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

4. $\left\{-\pi, -\frac{2\pi}{3}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$ 5. $\left\{\frac{1}{2}, \frac{5}{2}, \frac{7}{2}, \frac{11}{2}, \frac{13}{2}, \frac{17}{2}, \frac{19}{2}\right\}$ 6. $\left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \pi, \frac{3\pi}{2}\right\}$

7. $\left\{\frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \frac{13}{4}, \frac{17}{4}, \frac{21}{4}, \frac{25}{4}\right\}$ 8. $\left\{\frac{2}{3}, \frac{10}{3}, \frac{26}{3}, \frac{34}{3}\right\}$

X. 1.a) $x = 5\sqrt{2}, y = 5\sqrt{2}$ b) $x = 7, y = 7\sqrt{3}$ c) $x = 8\sqrt{3}, y = 4\sqrt{3}$ 2. 1764π 3. $A = \frac{s^2\sqrt{3}}{4}$, $108\sqrt{3}$

4. 88 in^2 5. 96π 6. 412 m^2 7. 157.5 cm^3

XI. 1. 245 2. Repeating numbers by 67,416,960 3. a. 479,001,600 b. 1320 c. 220

4. a. $x^3 + 3x^2 + 3x + 1$ b. $27x^3 - 54x^2 + 36x - 8$ c. $x^6 - 12x^4 + 48x^2 - 64$

d. $x^4 - 8x^3 + 24x^2 - 32x + 16$ e. $16x^4 + 96x^3 + 216x^2 + 216x + 81$

XII. 1. $y = (x-1)^2$ 2. $\frac{(x-4)^2}{4} + \frac{(y+1)^2}{1} = 1$

XIII. 1.a) $(2,-2\sqrt{3})$ b) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ c) $(\sqrt{3},1)$ d) $(1,-\sqrt{3})$

2.a) $(2\sqrt{13},0.983)\ (-2\sqrt{13},4.124)$ b) (5,2.214) (–5,5.356) c) $\left(\sqrt{6}, \frac{5\pi}{4}\right)\left(-\sqrt{6}, \frac{\pi}{4}\right)$

d) $(\sqrt{10},5.961)\ (-\sqrt{10},2.820)$

3.a) $r = 3$ b) $r = \frac{-2}{3\cos\theta - \sin\theta} = \frac{2}{\sin\theta - 3\cos\theta}$ c) $r = \frac{4}{1-\cos\theta}$ or $\frac{-4}{1+\cos\theta}$

4.a) $x^2 + y^2 - 4y = 0$ b) $y = \frac{\sqrt{3}}{3}x$ or $\sqrt{3}x - 3y = 0$ c) $4x^2 - 5y^2 - 36y - 36 = 0$

5.a) rose curve, 3 petals with length = 2, one curve on the interval $0 \le \theta \le \pi$

b) limaçon with inner loop, one curve on the interval $0 \le \theta \le 2\pi$

c) cardiod, one curve on the interval $0 \le \theta \le 2\pi$

AP Calculus I & II H
(BC Calculus)

Course Guidelines – Columbia High School

Mrs. Van Oort
vanoortna@egcsd.org
207-2133 Math Office

Description: Calculus BC is a college-level course taught over a full year in the high school. In order to be successful in this course, each student must have a solid background including geometry, trigonometry, and pre-calculus. The pace of this course is rapid and the student must be willing to dedicate 5-10 hours a week outside of the classroom to daily homework assignments, AP problem assignments, reading, and studying. During the year, we will cover the following topics in detail: functions, limits, continuity, differentiation, applications of the derivative, anti-differentiation, applications of the integral, derivatives and integrals of exponential & logarithmic functions, derivatives and integrals of trigonometric functions, series, convergence & divergence of series, and integration techniques.

EVERY STUDENT IN THIS CLASS IS EXPECTED TO TAKE THE ADVANCED PLACEMENT (AP) CALCULUS BC EXAM IN MAY AT A COST OF ABOUT $92 TO THE STUDENT!!

Supplies: 5 subject notebook (all 5 subjects…maybe more☺…will be used by May), loose-leaf paper, pen/pencil, graph paper.

Calculator: Graphing calculators are required for the AP exam. Graphing calculators will be made available in class, but the students' ability to use it is greatly improved by practice at home. The model used in class will be the TI-84 Plus, made by Texas Instruments. The TI-83 is similar. The TI-Nspire CAS is an advanced calculator that is allowed by the College Board and may also be used.

Grading Policy: Homework is an extremely important part of the class. Completing **daily** homework assignments is an ***integral*** part to being a successful calculus student. HW should be done neatly on *loose-leaf paper*, labeled with *your name* and *the assignment*. It will be checked and may be collected to count as a grade. *Homework might seem like a waste of time, but it teaches you content, time-management, and discipline – all of which you'll need in college. Homework time strongly predicts college success: Over half the students who do more than 10 hours of homework a week will get a four-year college degree; only about 16 percent of those doing less than three hours of homework a week will earn a bachelor's degree*[1].

AP problem assignments will also be given, collected, and counted as a grade. These assignments will be crucial to student learning as they will review topics previously covered and acclimate the students to the phrasing of AP questions.

Tests will be given approximately every three weeks and will be cumulative in nature. Tests will be given in-class, as a take-home, or some combination of the two methods. ***There will be no curves, shifts, or retests.***

ALL WORK IS EXPECTED TO BE THE STUDENT'S WORK ON ALL ASSIGNMENTS AND TESTS!!

[1] *James Rosenbaum* (2001) Beyond College for All; *Clifford Adelman* (1999) Answers in the Toolbox

Cell Phones/Electronic Devices: Silence and put away your cell phones and other electronic devices before entering the learning environment. Students are not to use or display any communication device, including cell phones, during assessments of any type, such as writing assignments, quizzes, tests, etc. Students observed using any prohibited communication device during these assessments (even if they have handed in their paper) will receive a **zero**.

Quarterly Grades: Quarterly grades are computed out of the total number of points assigned. For instance if there are 620 points given in the first quarter and you earn 580 points, your average is 580/620 = 94% = A.

Final Average:

Quarter 1	40%	Quarter 3	40%
Quarter 2	40%	Quarter 4	40%
Final Exam	20%	Final Exam	20%

HVCC Credit: This course is also offered for credit through Hudson Valley Community College. More information will soon follow, however, once the student has registered through HVCC, all questions and concerns about credit should be directed to the HVCC registrar's office.

CHS Policy Regarding the Dropping of Courses:
Half-year courses: No dropping of 20 week (1 semester) courses after ***five weeks***.

Excused Absences: When you are absent, you are responsible for getting missed assignments and notes. Tests and quizzes will be made up ***immediately***.

Unexcused Absences: If you skip class, you will not receive credit for any graded assignment completed that day. (This includes tests, quizzes, and projects as well as HW.)

Extra Help: If you are confused by a topic or would like extra help, please make attempts at getting this help. I will also be available before and after school. Please do not wait until it is too late to come for extra help!!!!

Expectations:

- Be on time or have a pass if you arrive after the bell. You will be given one warning. After your warning, you will be asked to serve a short detention after or before school.
- **Every day you will need to bring with you:** Your homework, a calculator, a writing utensil, and your binder. You may leave your book at home.
- Be **courteous and respectful** to your classmates and anyone else in the room.
- **No food or drink** allowed in the classroom.
- Please try to use the drinking fountain and bathroom between classes.